Examiners' Report

Principal Examiner Feedback

January 2023

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 02

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2023
Publications Code 4PM1_02_2301_ER

January 2023 Pearson Edexcel International GCSE Further Pure Mathematics (4PM1) paper 02

Principal Examiner Feedback

Introduction

On the whole, some candidates were well prepared for this examination with some very impressive scores. However, there wer4e also some candidates who were clearly unprepared and were unable to perform even the most fundamental and routine mathematical procedures.

The unstructured questions caused the greatest problems for candidates in this paper.

Question 1

This question was a simple introduction to this examination for candidates, and virtually all managed to obtain at least half the available marks here.

(a) They found solving this linear inequality very straightforward and most gained both marks. There were a very small minority who changed the inequality to an equals sign, losing both marks due to an incorrect process.

(b) Most candidates could solve the quadratic equation $3x^2 - 8x - 3 < 0$, usually by factorising to find the critical values required. This gained them the first two marks. Many then went onto identifying and writing the correct range as an inequality, although the correct region was not seen as often as it ought to have been. A common incorrect answer was $x < -\frac{1}{3}, x < 3$. It was disappointing to see how often this was presented as a solution. Centres should bear this in mind when teaching quadratic inequalities.

(c) This was not answered well by the majority of candidates. They could put their answers together from parts (a) and (b) to give a combined correct range. Some candidates seem to guess an answer here and give little thought to a sensible answer or if their answer would be valid.

Question 2

Generally the majority of candidates understood that applying the Cosine rule to the triangle and solving the resultant quadratic equation to obtain a solution which would allow $(2x + 1)$, $(2x + 4)$ and $(x + 2)$ to be consistent with a lengths of the triangle and the given angle was what was required. There were however, many algebraic errors to produce the correct three term quadratic. We did see just a few attempts to use sine rule as well.

Centres should note carefully that full working must always be shown. We credit correct roots following a correct quadratic equation using a calculator. We do not however, check roots from incorrect quadratics which have been solved using a calculator. Therefore, the general principle should always be: show working every time.

Some candidates having found and solved the correct quadratic equation then gave us both roots, or even $8 \pm 5\sqrt{3}$ which could not score the final mark as a negative length is not feasible.

<u>Question 3</u>

Considering the parts (a) and (b) are standard questions that have appeared on virtually every 4PM1 examination series, it was surprising to note how many candidates were unable to find the values of *A*, *B* and *C* correctly.

(a) Having factorised the 8 successfully, most realised that they needed to divide $\frac{10}{8}$ by 2. Some multiplied by 2, and some even did not remove the *x* and left the expression erroneously as $8\left(x+\frac{10}{16}x\right)^2+\ldots$

(b) Many candidates either extracted the required values form their work in part (a) for which full follow through credit was given, or differentiated to find a minimum value of *x* and used that to find the minimum value of f(*x*)

Parts (c) and (d) were answered very well indeed and were the source of six valuable marks, particularly for less able candidates with virtually every candidate managing to find the required coordinates accurately and correctly.

(e) This part of the question clearly demonstrated that many candidates do not have any visual awareness of linear or quadratic functions. We saw only a few fully correct sketches with both the line and curve drawn in the correct places. We were not strict on either and as long as the line crossed the positive *y*-axis and had a positive gradient we awarded the mark, and as long as the curve had a minimum below the x-axis we also awarded the mark. Despite this generosity, we seldom awarded both marks, and even then, it was for some very poorly drawn sketches.

Question 4

Product rule was not a major issue for this question, although some did not recognise that differentiation was required. A few students gave $'-\cos x'$ as the derivative of sin x. Because the differentiation was so simple, for the M mark we insisted that differentiation of sin x had to be correct, and only allowed minor latitude on the differentiation of x^3. Having said that, this was another question where the majority of candidates scored at least 3 marks.

There was a fair number of students who did not maintain exact values and proceeded to finding the equation of the tangent using decimals. Clearly, there is a level of discomfort working with numbers in terms of π. We gave credit for approximations but in future series, we will be careful to insist on exact values used throughout.

Some candidates substituted 90° into their $\frac{dy}{dx}$ rather than the required $\frac{\pi}{2}$. We could not give this any credit.

The most successful candidates used $y-y_1=m(x-x_1)$ to find the equation of the line. This method is slightly more reliable as the M mark is automatically scored with a correct substitution, whereas those candidates using $y=mx+c$ some manipulation was required to find a value for c before any mark could be awarded.

Question 5

(a) This part of the question was very well answered and was a good source of marks for almost every candidate. Almost all picked up the first mark by finding AC correctly using Pythagoras' Theorem. Most then went onto find the required length of h correctly. The most popular method was to use the tangent ratio, but there were other correct methods seen, sine rule for example. A small number of candidates had issues in rearranging or did not calculate or know the value of $\tan 30^\circ$.

(b) This part was not answered well by the majority of candidates. In most cases they could not interpret the diagram to realise how they would find the size of the angle *EFO* in the first place. Most could interpret the ratio given and used it to find $\frac{12}{5} = 2.4$ but then did not do anything correctly with this value that would have got them to the answer.

There were two strategies a candidate could use to find the required angle:

Method 1

Some realised that they could use this value of 2.4 cm to find the mid-point of *AD* to *F* and then use Pythagoras to find the length of *OF*. Those that managed this step this usually went onto score all the marks in this part by using the tan ratio correctly for the final angle.

Method 2

A very good alternative methods employed [which did not require the finding of the length 3.6 cm] was using cosine rule to find *OF*. Those who realised they could do this, usually went onto score all the marks. Students should be encouraged to annotate their diagrams and to draw small thumbnail sketches of the triangles in question so they can see what they need to work out to answer the question.

<u>Question 6</u>

Generally, this question was not very well answered at all because candidates did not understand that dividing terms in geometric series yields the common ratio.

(a) Setting up a correct equation to find *p* was difficult for many in this part. There were a minority who realised they needed to divide U_2 by U_1 and equate this equal to U_3 divided by U_2. It was clear from some very complicated and needless manipulations that some candidates had many attempts before figuring out what they needed to do. Many of those that set up the correct equation were able to get to the required three term quadratic and solve it by factorising to get the two values for *p*. Although when solving their quadratic by factorising many used the approach where they break up the term in *x* and factorise in 2 halves. However, this led to numerous sign errors and students often incorrectly tried to force their quadratic to factorise in this way.

The most common error in part (a) was to treat the sequence as arithmetic rather than geometric, so subtracting and equating the terms.

(b) Many used the wrong value of p in this part, although of those who managed to get this far in the question, we gave credit to an attempt to find values for r and a irrespective of the value used for p for the first M mark. However, many used $p=-\frac{1}{2}$ resulting in a common ratio greater than 1 thus achieving no further marks in this question. It seemed that students recalled something had to be between 1 and – 1 so used the p value that fitted this constraint rather than recalling it was the common ratio this applied to. Many used both values of p to find corresponding values of r and then realised they should only use $r=\frac{3}{5}$ and then found the first term with the correct value of p to apply the sum to infinity formula. This was a small minority of cases though.

Question 7

(a) Candidates appreciated that the product rule was required here and there were many correct answers to this part of the question. Those who could find the required derivative correctly mostly went on the obtain the required result. The errors we saw were largely the result of careless work such as;

$e^{2x} \Rightarrow 2e^{x}$, $\sin 2x \Rightarrow 2\cos x$ etc., all avoidable errors with a little care.

However, many candidates scored all four marks available here.

(b) A number of candidates did not even attempt this part and the majority who did scored 2 marks for a correct second derivative, or possibly 3 marks if they managed to simplify to $\frac{d^2y}{dx^2}=-8e^{2x}\sin 2x$ which was really a key result in obtaining the required result.

The successful candidates then proceeded to show that $4\frac{dy}{dx}-8y$ was also equal to $-8e^{2x}\sin 2x$ and thus having concluded these were equal scored full marks in this part. This was a rare sight however.

Question 8

This was an unstructured question in which candidates needed to formulate a strategy in order to solve it. Probably because of this lack of direction and structure a small number of candidates did not even bother to attempt this question at all and left a completely blank sheet. Centres should impress on their students the need to start questions such as these with standard techniques that will almost certainly yield some marks. There were enough hints and information in the stem of the question to give concrete direction.

Most candidates were able to start by scoring B1B1 for correct expressions of sum and product. A significant minority however, gave sum as $-4k\sqrt{2}$ or $4k\sqrt{2}x$ and the product was often given as – 1. A few candidates were confused about what to do, they set discriminant to zero and solved for *k*.

Many candidates had correct algebra on $\alpha^2+\beta^2$ and substituted their expressions of sum and product, set this = 66 and then solved correctly for *k*. We did see some clumsy algebraic manipulation and so some candidates lost marks while simplifying the equation incorrectly, hence leading to an incorrect *k* value.

For the rest of the question, a majority candidates were able to score M1A1 for writing down the correct algebra on $\alpha^3+\beta^3$. The most common correct form seen was given as $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ which is clearly well known. The last two marks in this question proved to be challenging to obtain. Those who worked out the correct *p* value often used the Method 1 in the mark scheme.

A small number of candidates used Method 2 to find the value of α and β via solving simultaneous equations, then substituted the values to $\alpha^3+\beta^3$, but candidates rarely seen to obtain the correct *p* value via Method 2 as the algebra defeated most. Of the small number of candidates who worked the $280\sqrt{2}$, some did not state *p* = 280 and unfortunately lost the last A mark.

Question 9

This question from the topic 'modelling with rates of change' was answered understood and solved very successfully by a significant minority of candidates who demonstrated full understanding of using rates of change as part of the chain rule of differentiation. Other candidates could only score two or three marks by stating $\frac{\mathrm{d}A}{\mathrm{d}t} = 0.45$ and usually $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ with x = 8. Some candidates had difficulties expressing the total surface area of the cube and as such could not achieve either an expression for A or the derivative with respect to x.

An incorrect expression for the total area A also triggered the loss of the method mark offered for correctly finding the value of x that was required to be substituted in the chain of derivatives. The candidates who fail to write a correct expression for the chain of derivatives leading to $\frac{\mathrm{d}V}{\mathrm{d}t}$ consequently lost the last 3 marks out of the total of seven as the substitution was dependant on a correct chain rule. The candidates who managed to write it correctly used either a direct explicit form $\left[\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}A} \times \frac{\mathrm{d}A}{\mathrm{d}t}\right]$ or completed in two stages

$$\left[\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{\frac{\mathrm{d}A}{\mathrm{d}x}} \times \frac{\mathrm{d}A}{\mathrm{d}t} \quad \textbf{AND} \quad \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} \times \frac{\mathrm{d}V}{\mathrm{d}x}\right].$$

Question 10

The question was well structured to allow marks to be achieved from implementation of formulae, algebraic manipulation of trig functions by way of a fairly straightforward proof and demonstration of applying trig functions to a range of angles, but overall, the performance of candidates was disappointing.

(a) A significant number of candidates failed to attempt the question despite the first five marks being a simple application of basic trig relationships and indeed, these are standard proofs that we have set many times. A number of candidates failed to recognise that the steps in a 'show that' questions need to be clear and generally in part a (ii) the first M was achieved but the existence of a second step – the explicit substitution of $\left(1-\cos^2\theta\right)$ into $\cos 2\theta = \cos^2\theta - \sin^2\theta$ was not clearly shown leading to a loss of 2 marks.

(b) In part (b) there was a worrying lack of understanding or engagement with algebraic manipulation of trig functions. A significant number of candidates failed to attempt this part despite achieving full marks in part (a). Many candidates were able to start this part of the question and score the first M mark by writing $\sin 2\theta - \tan\theta = 2\sin\theta\cos\theta - \frac{\sin\theta}{\cos\theta}$ but could not proceed further than that. The small minority of candidates who made the second correct step went on mostly to achieve full marks demonstrating that the lack of success was more likely due to candidate‘s lack of exposure to manipulation of trig functions rather than difficulty of the question. Once again, there is a need for candidates to understand that a ‘show that’ question implies every stage must be clearly shown.

(c) This part was completed by some candidates who did not attempt part (b) or who gained only the first M mark. Most attempts demonstrated an understanding of the limits given and showed the steps required to reach the solution. A good many candidates failed to attempt this part probably because they had been intimidated by the trig manipulation in part (b) and failed to recognise the stand-alone nature of this part.

Some candidates used the expression from part (b) and some started all over again. A common error appearing all too frequently was the casual cancellation throughout of sin x leading to the inevitable loss of solutions. The B mark was awarded for an angle of 180° and this was very commonly missing. In fact the most common marking pattern in this part of the question was B0M1A1A0 because very few candidates indeed managed to find all correct angles.

Here, the successful candidates extended their range by writing 0° < 2x < 720°. Every student who did this went on to get all 3 marks. Rewriting the range of the function when solving $\cos 2x = 0$ is key to getting all the solutions.

Question 11

The vast majority of candidates found this question very difficult. Unsurprisingly a large number of candidates made no or little attempt to answer this question although whether this was through either complexity or time management it is impossible to deduce.

(a) (i) This was well answered by all, almost all picked up B1 for y = 3.

(a) (ii) Most got both marks here for showing that x = ln2. However not all showed the full method here, losing both marks. Centres must emphasise the importance of showing every step in a 'show that' question. Candidates benefitted from dealing with dealing with $\frac{1}{2}\ln 4$, and few showed working that could be evidence of a correct application.

(b) Many could differentiate the curve to get the first method mark. Some errors were then seen when substituting ln 2 into this differential function to get the perpendicular gradient for the line equation. Some did not find or even use the perpendicular gradient after finding –8 correctly, and if they used –8 as the gradient in their line equation losing the final two marks in this part was inevitable. There were some excellent answers seen here as well though, differentiating the exponential and finding the equation of the normal easily. Others tried to substitute 3 in for the y value instead of 0 in the equation of the normal. Many who attempted this question did progress to the correct result, although too many still give their answer in decimal form, not appreciating that exact form is not only acceptable but is preferable.

(c) Those candidates who got this far into the question integrated the curve between the correct limits for the area under the curve. This integration was usually done correctly and the majority show the substitution of their limits following the instruction in the question to 'use calculus'. The area of the triangle was not so well done. There were two methods, using $\frac{1}{2}\times b\times h$ (by far the simpler method) and integration, both methods being seen many times though far too often unsuccessfully. Unfortunately, those that tried to integrate, in many cases, incorrectly combined their integration into the integration under the curve. Very few explicitly showed the substitution of limits step and this must be encouraged in future series. There were also a few very good solutions seen here, candidates able to identify that they needed both areas and add them together, changing the sign of their area of the triangle if they used integration and finding the correct final answer. Very few actually illustrated the area they were calculating on the graph provided, which is important for success in these questions.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom