Mark Scheme (Results)

January 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where } |pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where} |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."**Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Paper 2		
Question number	Scheme	Marks
1 a	$9x < 6 \Rightarrow x < ...$	M1
	$x < \frac{2}{3}$	A1 (2)
b	$(3x+1)(x-3) < 0$	M1
	$x = -\frac{1}{3}$ $\quad x = 3$	A1
	$-\frac{1}{3} < x < 3$	M1 A1 (4)
c	$-\frac{1}{3} < x < \frac{2}{3}$	B1ft (1)
		Total 7 marks

Part	**Mark**	**Notes**
(a)	M1	For a complete method to find a value for *x* They must obtain a value for *x* with at most one processing error. The inequality must be correct in this part of the question.
	A1	For $x < \frac{2}{3}$ Accept awrt 0.67
(b)	M1	For attempting to factorise or otherwise solve the given quadratic using any method. If there is no method, [use of a calculator] then both roots must be fully correct for evidence of this mark. See general guidance for the definition of an attempt. Accept <, >, = or even no sign at all for this mark.
	A1	For **both** correct critical values. $x = -\frac{1}{3}$ $\quad x = 3$ Accept awrt -0.33
	M1	For a correct **inside** region using their values $'-\frac{1}{3}' < x < '3'$
	A1	For $-\frac{1}{3} < x < 3$
(c)	B1ft	For $'-\frac{1}{3}' < x < '\frac{2}{3}'$ Ft their values from parts (a) and (b), providing they are inequalities. **Do not** follow through an equals sign given in part (a). Allow recovery for a fully correct answer seen.

Question number	Scheme	Marks
2	$(2x-1)^2=(2x+4)^2+(x+2)^2-2(2x+4)(x+2)\cos 60^\circ$	M1
	$=x^2-16x-11=0$	M1
	$x=\dfrac{16\pm\sqrt{16^2+4\times 11}}{2}=8\pm 5\sqrt{3}$	M1
	[If $x=8-5\sqrt{3}$ then $2x-1$ is negative, so] $x=8+5\sqrt{3}$	A1 (4)
		Total 4 marks

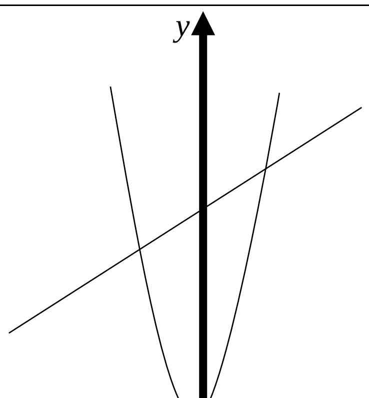
Mark	Notes
M1	For the correct use of a correct cosine rule. Either $(2x-1)^2=(2x+4)^2+(x+2)^2-2(2x+4)(x+2)\cos 60^\circ$ or $\cos 60^\circ=\dfrac{(2x+4)^2+(x+2)^2-(2x-1)^2}{2\times(2x+4)\times(x+2)}$
M1	For simplification of their expression to a 3TQ They must reach as a minimum; $Px^2\pm Qx\pm R=0$ where P , Q and R are non-zero constants Accept the terms in any order. Accept even for example $x^2-16x=11$
M1	For an attempt to solve their 3TQ using any method. See General Guidance for the definition of an attempt. **NOTE:** If their 3TQ is **incorrect** and they do **NOT** show us a valid method to solve it, and two roots just appear, **this is M0**.
A1	For $x=8+5\sqrt{3}$ **only** (must reject $x=8-5\sqrt{3}$)

Question number	Scheme	Marks
3 a	$8\left(x^2+\frac{10}{8}x\right)-3$	M1
	$8\left(x+\frac{5}{8}\right)^2-\frac{200}{64}-3=8\left(x+\frac{5}{8}\right)^2-\frac{49}{8}$	M1
	So $A = 8$ $B = \frac{5}{8}$ $C = -\frac{49}{8}$	A1 (3)
b (i)	$x=-\frac{5}{8}$	B1 ft
(ii)	$\text{f}(x)_{\text{min}}=-\frac{49}{8}$	B1 ft (2)
c	$8\left(x+\frac{5}{8}\right)^2-\frac{49}{8}=0\Rightarrow x=-\frac{3}{2}\quad x=\frac{1}{4}$	M1 A1 (2)
d	$8x^2+10x-3=2x+13\Rightarrow 8x^2+8x-16=0$ oe	M1
	$x^2+x-2=0=(x+2)(x-1)\Rightarrow x=-2,\ x=1$	M1 A1
	$y=9\quad y=15$ Coordinates are $(-2,9)$ and $(1,15)$	A1 (4)
e	Correct curve or line Correct curve and line	B1 B1 (2)
		Total 13 marks

Part	Mark	Notes
(a)	M1	For factorising in the form $8\left(x^2+bx\right)+c$ or $8\left(x^2+bx+d\right)$
	M1	For completing the square in the form $8\left(x+\frac{1}{2}b\right)^2+e$ [see GG]
	A1	For $A = 8$ $B = \frac{5}{8}$ $C = -\frac{49}{8}$ Accept these listed or embedded.
	ALT	
	M1	For expanding $\left[\text{f}(x)\right]=A(x+B)^2+C=Ax^2+2ABx+\left(AB^2+C\right)$ This must be correct for this mark
	M1	Equates coefficients: $Ax^2+2ABx+\left(AB^2+C\right)=8x^2+10x-3$ $\Rightarrow A=8,\quad 2AB=10,\quad AB^2+C=-3$ At least two out of three must be correct.
	A1	For $A = 8$ $B = \frac{5}{8}$ $C = -\frac{49}{8}$

(b) (i)	B1ft	For $x = -\frac{5}{8}$ ft their $-B$ Allow differentiation: $\frac{\mathrm{d}y}{\mathrm{d}x} = 16x + 10 = 0 \Rightarrow x = -\frac{10}{16}$ which must be correct.
(ii)	B1ft	For $\mathrm{f}(x)_{\min} = -\frac{49}{8}$ ft their C If they differentiate, allow a ft from their differentiation.
	Incorrect or no labelling of parts. If the responses are labelled incorrectly, mark as labelled. If there is no labelling, treat the first as (i) and the second as (ii) and mark accordingly.	
(c)	M1	For setting the given f(x) = 0 and solving using any method. See General Guidance.
	A1	For $x = -\frac{3}{2}$ and $x = \frac{1}{4}$
(d)	M1	For setting $8x^2 + 10x - 3 = 2x + 13$ and forming a 3TQ which as a minimum must be $8x^2 + 8x \pm C$ where C is a constant, or any simplification, e.g., $x^2 + x \pm K$ where K is a constant.
	M1	For attempting to solve their 3TQ. See General Guidance.
	A1	For **both** $x = -2,\ x = 1$
	A1	For $(-2, 9)$ and $(1, 15)$ Accept $x = -2,\ y = 9$ and $x = 1,\ y = 15$ paired correctly.
	ALT – solves the equation in y	
	M1	Substitutes $x = \frac{y-13}{2}$ into $y = 8x^2 + 10x - 3$ and forms a 3TQ which as a minimum must be of the form $2y^2 - 36y \pm X \quad X \neq 0$ $\Rightarrow y = 8\left(\frac{y-13}{2}\right)^2 + 10\left(\frac{y-13}{2}\right) - 3 \Rightarrow 2y^2 - 36y + 75 = 0$
	M1	For attempting to solve their 3TQ. See General Guidance.
	A1	For **both** y= 9 and y = 15
	A1	For $(-2, 9)$ and $(1, 15)$ Accept $x = -2,\ y = 9$ and $x = 1,\ y = 15$ paired correctly.
(e)	B1	For correct curve (intersections with x-axis are at (– 1.5, 0) and (0.25, 0)) **or** line (Intersections are (– 6.5, 0) and (0, 13) drawn. **We do not need to see any of these points marked. These are for guidance only.** The question asks for a sketch, accept a reasonable attempt. • Accept a positive quadratic curve with the minimum point below the x-axis, and one branch either side of the y-axis. • Accept a straight line with a positive gradient where the intersection with the y-axis is positive.
	B1	For correct curve **and** line drawn

SKETCH OF CURVE AND LINE

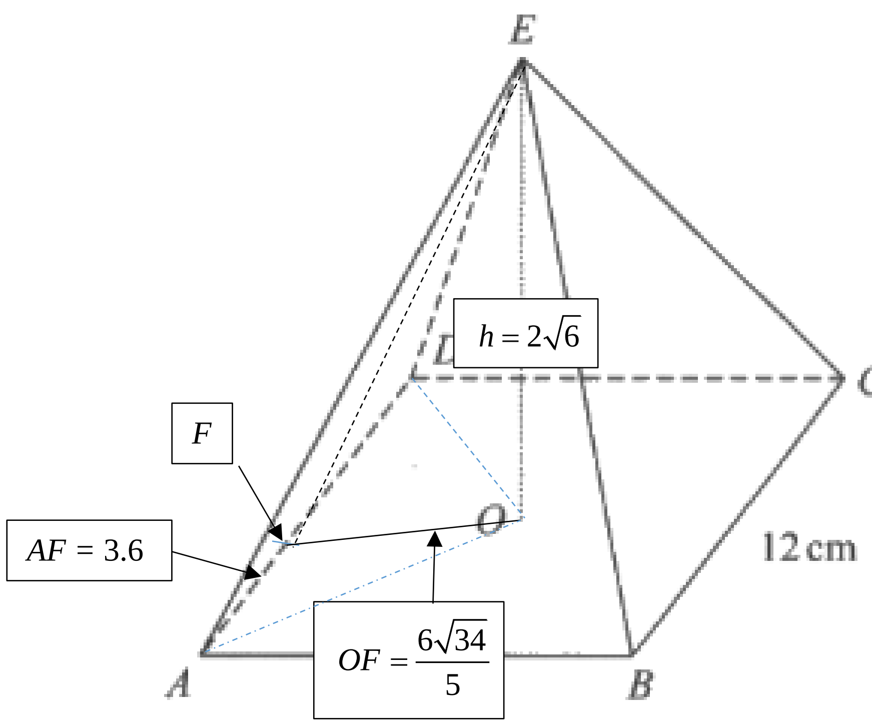


Question number	Scheme	Marks
4	When $x=\dfrac{\pi}{2}$ $\quad y=\dfrac{\pi^3}{8}$ So $\left(\dfrac{\pi}{2},\dfrac{\pi^3}{8}\right)$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=3x^2\sin x+x^3\cos x$	M1 A1 A1
	When $x=\dfrac{\pi}{2}$ $\quad \dfrac{\mathrm{d}y}{\mathrm{d}x}=3\left(\dfrac{\pi}{2}\right)^2\sin\left(\dfrac{\pi}{2}\right)+\left(\dfrac{\pi}{2}\right)^3\cos\left(\dfrac{\pi}{2}\right)=\left[\text{'}\dfrac{3\pi^2}{4}\text{'}\right]$	M1
	$y-\dfrac{\pi^3}{8}=\dfrac{3\pi^2}{4}\left(x-\dfrac{\pi}{2}\right)$	M1
	$y=\dfrac{3}{4}\pi^2x-\dfrac{1}{4}\pi^3$	A1
		Total 7 marks

Mark	Notes
Note: In this question, all substitution of angle values must be in Radians only.	
B1	For obtaining $y=\dfrac{\pi^3}{8}$ [allow $\left(\dfrac{\pi}{2}\right)^3$ and also awrt $y = 3.88$]
M1	For an attempt to use the product rule. The definition of an attempt is as follows: • There must be a correct attempt to differentiate both terms. $\sin x \Rightarrow \cos x \quad x^3 \Rightarrow ax^2$ where $x \neq 0$ • The correct formula must be used. i.e., it must be a sum of their two terms.
A1	At least one term must be correct. Either $3x^2 \sin x$ or $x^3 \cos x$
A1	For $3x^2 \sin x + x^3 \cos x$ oe Ignore any subsequent simplification once you have seen the correct answer – even if the simplification is incorrect.
M1	For substitution of $\dfrac{\pi}{2}$ into their $\dfrac{dy}{dx}=\left['\dfrac{3}{4}\pi^2'\right]$ provided it is a changed expression. Allow a value of awrt 7.4(0) **NOTE:** You must see a full substitution of $\dfrac{\pi}{2}$ into their $\dfrac{dy}{dx}$ if their expression for $\dfrac{dy}{dx}$ is incorrect.
M1	For a correct method for finding the equation of a line using their value of y, dy/dx and the given x [allow $x = 1.57…$]. This must be applied correctly. Either uses the formula with their values, or if uses $y = mx + c$ they must reach a value for c before this mark can be awarded. Do **not** allow processing errors to find the value of c
A1	For $y=\dfrac{3}{4}\pi^2 x-\dfrac{1}{4}\pi^3$ Allow $y = 7.4(0)x - 7.75$ or better values. Do not allow a mixture of decimals and exact values.

Question number	Scheme	Marks
5 a	$OC = \frac{1}{2}\sqrt{12^2 + 12^2} = \left[6\sqrt{2}\right]$ or $AC = \sqrt{12^2 + 12^2} = \left[12\sqrt{2}\right]$	M1
	$h = EO = 6\sqrt{2} \times \frac{\sqrt{3}}{3} = 2\sqrt{6}$ oe	M1 A1
		(3)
b	Midpoint of *AD* to *F* = $\left(6 - \frac{12}{1+4}\right)$ [= 3.6]	M1
	$OF = \sqrt{6^2 + 3.6^2} = \frac{6\sqrt{34}}{5}$	M1
	$\tan\theta = \frac{2\sqrt{6}}{\frac{6\sqrt{34}}{5}} = 35^\circ$	M1 A1
		(4)
		Total 7 marks

USEFUL SKETCH



Part	Mark	Notes
(a)	M1	For using a correct Pythagoras theorem to find either *OC*, *OA* $\left(6\sqrt{2}\right)$ or *AC* $\left(12\sqrt{2}\right)$
	M1	For using the correct trigonometry (tan ratio): $h='6\sqrt{2}'\tan 30^{\circ}=\left[2\sqrt{6}\right]$ or equivalent. Eg $\tan 30=\dfrac{h}{6\sqrt{2}}\Rightarrow h=....$
	A1	For $2\sqrt{6}$ oe
(b)	M1	For correct expression for the midpoint of *AD* to *F* $\left(6-\dfrac{12}{1+4}\right)$ or 3.6 oe seen
	M1	For the correct use of Pythagoras' to find *OF* $\dfrac{6\sqrt{34}}{5}$ oe [6.997..] ft their 3.6
	ALT	
	M1	Finds the length *OF* using cosine rule. $OF=\sqrt{2.4^2+\left(6\sqrt{2}\right)^2-2\times 2.4\times 6\sqrt{2}\times\cos 45}=\left[\dfrac{6\sqrt{34}}{5}\right]$ OR $OF=\sqrt{9.6^2+\left(6\sqrt{2}\right)^2-2\times 9.6\times 6\sqrt{2}\times\cos 45}=\left[\dfrac{6\sqrt{34}}{5}\right]$ OR $\cos 45^{\circ}=\dfrac{9.6^2+\left(6\sqrt{2}\right)^2-OF^2}{2\times 9.6\times 6\sqrt{2}}\Rightarrow OF=...$ oe [6.997..]
	M1	For the correct evaluation of their cosine rule. $\left(\dfrac{6\sqrt{34}}{5}\text{ oe}\right)$ Accept awrt 7.00 [6.997142…]
	dM1	For the correct use of $\tan\theta=\dfrac{'EO'}{'OF'}$ This mark is dependent on both previous M marks. They must have a valid method to find *OF* for the award of this mark.
	A1	For 35(°) or better (Calculator value is 34.997…°.)
	Note: There are other methods – if unsure, send to Review.	

Question number	Scheme	Marks
6 a	$r = \dfrac{q(2p-3)}{q(2p+3)} = \dfrac{q(2p+3)}{q(4p+1)}$	M1
	$(4p+1)(2p-3) = (2p+3)^2$	dM1
	$2p^2 - 11p - 6 = 0$	A1
	$(2p+1)(p-6) = 0$	M1
	$p = -\dfrac{1}{2}$ or $p = 6$	A1 (5)
b	When $p = 6$ $r = \dfrac{3}{5}$ and $U_1 = q\left(4 \times '6' + 1\right) = \left(25q\right)$	M1
	$S_\infty = \dfrac{25q}{\frac{2}{5}} = 250 \Rightarrow q = 4$	dM1 A1 (3)
		Total 8 marks

Part	Mark	Notes
(a)	M1	For $r=\dfrac{\text{3rd term}}{\text{2nd term}}=\dfrac{\text{2nd term}}{\text{1st term}}$
	dM1	For attempting to remove the denominators/simplifying the expression to attempt to obtain a 3TQ. Allow one processing error. Allow a minimally acceptable $4p^2 \pm Xp \pm Y=(0)$ or $2p^2 \pm Pp \pm Q=(0)$ $\quad X,Y,P,Q\neq 0$ **Note:** This mark is dependent on the previous M mark.
	A1	For obtaining $2p^2-11p-6=(0)$ or equivalent. Their working will give them $4p^2-22p-12=(0)$ before further simplification
	M1	For a valid attempt to solve the 3TQ. Note this is an independent mark for their 3TQ. See General Guidance. The 3TQ must have come from some attempted manipulation involving $q(2p-3)$, $q(2p+3)$ and $q(4p+1)$
	A1	For $p=-\dfrac{1}{2}$ or $p=6$
(b)	M1	For finding: • A value of r for their p. Allow any value of r $\quad r\neq 0$ E.g. $r=\dfrac{(2\times '6'-3)}{(2\times '6'+3)}=....$ $\left(\text{Note: for } p=-\dfrac{1}{2} \quad r=-2\right)$ • A value for U_1 $U_1=q(4\times '6'+1)=...$ $\left(\text{Note: for } p=-\dfrac{1}{2}, U_1=-q\right)$ You may see a = 100 after later working to find the sum to infinity. If their values of p are incorrect, we **must** see working here. **NOTE:** This is a B mark in Epen.
	dM1	For use of $S_\infty=\dfrac{a}{1-r}=250$ with their r and their U_1 provided $\lvert r\rvert<1$ If they use the formula for the sum to infinity on an $\lvert r\rvert>1$ withhold this mark even if they use it on one valid and one invalid attempt. **Note:** This mark is dependent on the previous M mark.
	A1	For $q=4$

Question number	Scheme	Marks
7 a	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{2x}\cos 2x - 2\mathrm{e}^{2x}\sin 2x$	M1 A1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2y - 2\mathrm{e}^{2x}\sin 2x$ *	A1 cso (4)
b	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\frac{\mathrm{d}y}{\mathrm{d}x} - 4\mathrm{e}^{2x}\sin 2x - 4\mathrm{e}^{2x}\cos 2x$	M1 A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2\left(\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\right) - 4y$	dM1ddM1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\frac{\mathrm{d}y}{\mathrm{d}x} - 8y$ *	A1 cso (5)
		Total 9 marks

Part	Mark	Notes
(a)	M1	For an attempt to use the product rule. The definition of an attempt is as follows: • There must be a correct attempt to differentiate both terms. $\cos 2x \Rightarrow \pm 2\sin 2x \quad \mathrm{e}^{2x} \Rightarrow 2\mathrm{e}^{2x}$ where $x \neq 0$ • The terms must be added The correct formula must be used.
	A1	For one term correct
	A1	For both terms correct
	A1 cso	For obtaining the given result with no errors seen.
(b)	**Method 1**	
	M1	For attempting to differentiate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\mathrm{e}^{2x}\cos 2x - 4\mathrm{e}^{2x}\sin 2x - 4\mathrm{e}^{2x}\sin 2x - 4\mathrm{e}^{2x}\cos 2x$ For this mark accept either: $\left(4\mathrm{e}^{2x}\cos 2x - 4\mathrm{e}^{2x}\sin 2x\right)$ **OR** $-4\mathrm{e}^{2x}\sin 2x - 4\mathrm{e}^{2x}\cos 2x$
	A1	For the correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ (This is an M mark in Epen)
	dM1	For simplifying $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ to obtain $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -8\mathrm{e}^{2x}\sin 2x$ (A mark in Epen) **This mark is dependent on the first M mark**

	ddM1	For using the substitution $-2e^{2x}\sin 2x = \frac{dy}{dx} - 2y \Rightarrow -8e^{2x}\sin 2x = 4\frac{dy}{dx} - 8y$ **This mark is dependent on BOTH previous M marks.**
	A1cso	For obtaining the given result with no errors seen. $\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$
	Method 2	
	M1	For attempting to differentiate $\frac{dy}{dx}$ to obtain $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4e^{2x}\sin 2x - 4e^{2x}\cos 2x$ For this mark accept either: $2\frac{dy}{dx}$ or $-4e^{2x}\sin 2x - 4e^{2x}\cos 2x$
	A1	For the correct $\frac{d^2y}{dx^2}$ **This is an M mark in Epen**
	dM1	For substituting $-4e^{2x}\cos 2x \Rightarrow -2y$ **This is an A mark in Epen**
	ddM1	For using the substitution $-4e^{2x}\sin 2x = 2\frac{dy}{dx} - 4y$
	A1	For obtaining the given result with no errors seen. $\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$
	Method 3 – Works form LHS and RHS together.	
	M1	For attempting to differentiate $\frac{dy}{dx}$ to obtain $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 4e^{2x}\cos 2x - 4e^{2x}\sin 2x - 4e^{2x}\sin 2x - 4e^{2x}\cos 2x$ For this mark accept either: $\left(4e^{2x}\cos 2x - 4e^{2x}\sin 2x\right)$ **OR** $-4e^{2x}\sin 2x - 4e^{2x}\cos 2x$
	A1	For the correct $\frac{d^2y}{dx^2}$ **This is an M mark in Epen**
	dM1	For simplifying $\frac{d^2y}{dx^2}$ to obtain $\frac{d^2y}{dx^2} = -8e^{2x}\sin 2x$ (A mark in Epen) **This mark is dependent on the first M mark**
	ddM1	Multiplies out the given expression in (b) $\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y = 4\left(2e^{2x}\cos 2x - 2e^{2x}\sin 2x\right) - 8\left(e^{2x}\cos 2x\right)$ $= 8e^{2x}\cos 2x - 8e^{2x}\sin 2x - 8e^{2x}\cos 2x$ $= -8e^{2x}\sin 2x$

		This mark is dependent on BOTH previous M marks.
	A1	For a conclusion. A simple # sign, 'shown', 'QED' or underlining is sufficient.

Question number	Scheme	Marks
8	$\alpha+\beta=4k\sqrt{2}$ and $\alpha\beta=2k^4-1$	B1 B1
	$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta \Rightarrow \left(4k\sqrt{2}\right)^2=66+2\left(2k^4-1\right)$	M1 A1
	$k^4-8k^2+16=0$	M1
	$\left(k^2-4\right)^2=0 \Rightarrow k=2$	M1 A1
	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$ **or** $\alpha^3+\beta^3=(\alpha+\beta)\left(\alpha^2+\beta^2-\alpha\beta\right)$	M1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ **or** $\alpha^3+\beta^3=(\alpha+\beta)(66-\alpha\beta)$	A1
	$\alpha^3+\beta^3=\left(8\sqrt{2}\right)^3-3\times31\times8\sqrt{2}=280\sqrt{2}$ **or** $8\sqrt{2}(66-31)=280\sqrt{2}$	M1
	p=280	A1 (11)
		Total 11 marks

Mark	Notes
B1	For **either** $\alpha+\beta=4k\sqrt{2}$ **or** $\alpha\beta=2k^4-1$
B1	For **both** $\alpha+\beta=4k\sqrt{2}$ **and** $\alpha\beta=2k^4-1$
M1	For the correct algebra on $\alpha^2+\beta^2$ (in any order) **and** substitution of their values of $\alpha\beta$ and $\alpha+\beta$ providing both sum and product are in terms of k. $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta \Rightarrow \alpha^2+\beta^2=\left('4k\sqrt{2}'\right)^2-2\left('2k^4-1'\right)$
A1	For obtaining $\left(4k\sqrt{2}\right)^2=66+2\left(2k^4-1\right)$ in any order.
M1	For simplifying to form a 3TQ in k^4 i.e., $4k^4-32k^2+64=0$ oe Accept as a minimum $4k^4-32k^2\pm Q=(0)$ $Q\neq0$
M1	For factorising or solving the 3TQ using any valid method. See General Guidance.
A1	For $k=2$ If they also give $k=-2$ withhold this mark.

Method 1	
M1	For expanding $(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ or $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)$
A1	For obtaining $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ or $\alpha^3 + \beta^3 = (\alpha+\beta)(66 - \alpha\beta)$ This must be such that $\alpha\beta$ and $\alpha+\beta$ can be substituted in directly.
M1	For substitution of $\alpha+\beta$ and $\alpha\beta$ for their positive value of *k* into a **correct** expansion of $\alpha^3+\beta^3$ **NOTE:** If they do not obtain $k = 2$, then full substitution of their numerical value for *k* into $\alpha+\beta$ **and** $\alpha\beta$ must be seen for the award of this mark. For example: $\alpha^3 + \beta^3 = \left(4\times\text{'their }k\text{'}\sqrt{2}\right)^3 - 3\times\left(2\times\text{'their }k^4\text{'}-1\right)\times\left(4\times\text{'their }k\text{'}\sqrt{2}\right)$
A1	For $p = 280$
Method 2	
M1	Finds the exact value of α and β Solves the equations $\left(\alpha\beta = 2k^4 - 1 \text{ and } \alpha+\beta = 4k\sqrt{2}\right)$ simultaneously to give a value for α and β $\alpha = \dfrac{31}{\beta} \Rightarrow \alpha+\beta = \dfrac{31}{\beta} + \beta = 8\sqrt{2} \Rightarrow \beta^2 - 8\sqrt{2}\beta + 31 = 0$ $\Rightarrow \beta = \ldots \quad \alpha = \ldots$
A1	For $\alpha = 1 + 4\sqrt{2} \quad \beta = -1 + 4\sqrt{2}$ OR $\beta = 1 + 4\sqrt{2} \quad \alpha = -1 + 4\sqrt{2}$
M1	Substitutes these values into $\alpha^3 + \beta^3 = p\sqrt{2}$ to find a value for *p* $\left(1+4\sqrt{2}\right)^3 + \left(-1+4\sqrt{2}\right)^3 = 24\sqrt{2} + 256\sqrt{2} = 280\sqrt{2} \Rightarrow p = \ldots$
A1	$p = 280$

Question number	Scheme	Marks
9		
	$\dfrac{dA}{dt} = 0.45$	B1
	$V = x^3 \Rightarrow \dfrac{dV}{dx} = 3x^2$	B1
	$A = 6x^2 \Rightarrow \dfrac{dA}{dx} = 12x$	B1
	$384 = 6x^2 \Rightarrow x = 8$	M1
	$\dfrac{dV}{dt} = \dfrac{dV}{dx} \times \dfrac{dx}{dA} \times \dfrac{dA}{dt}$	M1
	$\dfrac{dV}{dt} = 3x^2 \times \dfrac{1}{12x} \times 0.45 \left[= \dfrac{9x}{80} \right]$ oe	dM1
	When $x = 8$ $\dfrac{dV}{dt} = 0.9\,\text{cm}^3/\text{s}$	A1
		Total 7 marks

Mark	Notes
B1	For $\frac{\mathrm{d}A}{\mathrm{d}t}=0.45$ seen anywhere in their working Accept other letters, for example S for the area $\frac{\mathrm{d}S}{\mathrm{d}t}=0.45$
B1	For $\frac{\mathrm{d}V}{\mathrm{d}x}=3x^2$ Accept also other letters in place of x such as r for example.
B1	For $\frac{\mathrm{d}A}{\mathrm{d}x}=12x$ Accept also other letters in place of x such as r for example.
M1	For setting $384=6x^2$ and proceeding to a correct method leading to a value of x Award this mark when they obtain $x^2=64 \Rightarrow x=...$
M1	For a correct expression of the chain rule seen or implied. i.e., $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}A}\times\frac{\mathrm{d}A}{\mathrm{d}t}$ They may complete this in two stages. So you may see for example: $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{\frac{\mathrm{d}A}{\mathrm{d}x}}\times\frac{\mathrm{d}A}{\mathrm{d}t}$ **AND** $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}x}{\mathrm{d}t}\times\frac{\mathrm{d}V}{\mathrm{d}x}$
dM1	For substituting their values into a **correct** chain rule. $\frac{\mathrm{d}V}{\mathrm{d}t}=3(8)^2\times\frac{1}{12(8)}\times 0.45$ This mark is dependent on the previous M mark scored.
A1	For 0.9 (cm^3/s)

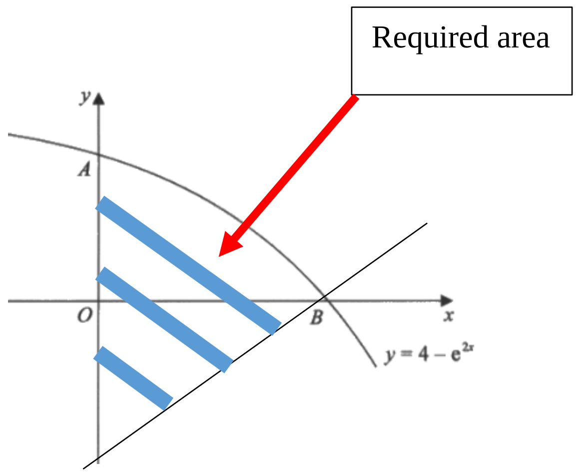
Question number	Scheme	Marks
10 a (i)	$\sin 2\theta = \sin\theta\cos\theta + \cos\theta\sin\theta = 2\sin\theta\cos\theta$ *	M1 A1 cso (2)
a (ii)	$\cos 2\theta = \cos^2\theta - \sin^2\theta = \cos^2\theta - (1-\cos^2\theta)$	M1 M1
	$= 2\cos^2\theta - 1$ *	A1 cso (3)
b	$\sin 2\theta - \tan\theta = 2\sin\theta\cos\theta - \dfrac{\sin\theta}{\cos\theta}$	M1
	$\sin\theta\left(2\cos\theta - \dfrac{1}{\cos\theta}\right)$	M1
	$\sin\theta\left(\dfrac{2\cos^2\theta - 1}{\cos\theta}\right) = \tan\theta\cos 2\theta$ *	dM1 A1cso (4)
c	$\tan x = 0$ so $x = 180$	B1
	$\cos 2x = 0$ so $2x = 90, 270, 450, 630$	M1
	$x = 45, 135, 225, 315$	A1 A1 (4)
		Total 13 marks

Part	**Mark**	**Notes**
(a)(i)	M1	For the correct use of: $\sin(A+B) = \sin A\cos B + \cos A\sin B \Rightarrow \sin 2\theta = \sin\theta\cos\theta + \sin\theta\cos\theta$ Allow any letter to be used for this mark.
	A1 Cso	For obtaining the given result $\sin 2\theta = 2\sin\theta\cos\theta$ or allow $\sin(\theta+\theta) = 2\sin\theta\cos\theta$
(a)(ii)	M1	For the correct use of: $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\Rightarrow \cos 2A = \cos^2 A - \sin^2 A$ or $\cos(A+A) = \cos A\cos A - \sin A\sin A$ Allow any letter to be used for this mark.
	M1	For use of $\sin^2 A + \cos^2 A = 1$ and substituting Allow any letter to be used for this mark.
	A1 cso	For obtaining the given result $\cos 2\theta = 2\cos^2\theta - 1$ or allow $\cos(\theta+\theta) = 2\cos^2\theta - 1$
(b)	M1	For correct use of $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

	M1	For factorising the resulting expression to give $\sin\theta\left(2\cos\theta-\dfrac{1}{\cos\theta}\right)$ **and** using a common denominator of $\cos\theta$ correctly to obtain $\sin\theta\left(\dfrac{2\cos^2\theta-1}{\cos\theta}\right)$ oe e.g. $\dfrac{\sin\theta}{\cos\theta}\left(2\cos^2\theta-1\right)$
	dM1	For use of $2\cos^2\theta-1=\cos 2\theta$ **Note:** This mark is dependent on both previous M marks
	A1 cso	For obtaining the given result
	ALT - Allow candidates to work from both sides. For example.	
	M1	For correct use of $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ $\sin 2\theta-\tan\theta=\tan\theta\cos 2\theta$ $\Rightarrow 2\sin\theta\cos\theta-\dfrac{\sin\theta}{\cos\theta}=\dfrac{\sin\theta}{\cos\theta}\left(2\cos^2\theta-1\right)$
	M1	For using a common denominator on the LHS to give: $\dfrac{2\sin\theta\cos^2\theta-\sin\theta}{\cos\theta}=\dfrac{\sin\theta\left(2\cos^2\theta-1\right)}{\cos\theta}$
	dM1	Multiplies or factorises one side to make LHS = RHS For example; $\dfrac{2\sin\theta\left(\cos^2\theta-1\right)}{\cos\theta}=\dfrac{2\sin\theta\left(\cos^2\theta-1\right)}{\cos\theta}$
	A1 cso	Concludes RHS = LHS therefore shown. Note there must be no errors seen. If unsure, send to Review.
(c)	Some candidates are starting again without using the given expression in (b) and are getting their angles from cos *x* rather than cos 2*x* Allow the use of θ or any other variable throughout this part of the question. As a general principle of marking this part: • B1 – For only 180° seen. If 0° is included, then B0. • M1 – For either $2x=90,(270,450,630)$ **OR** $x=45^\circ$ or 135° or 225° or 315° or any other correct angle in or out of range. • A1 – For any two correct angles apart from 180° • A1 – For all four correct angles, apart form 180°	
	B1	For $x=180$ If $x=0$ included then B0
	M1	For obtaining $2x=90,(270,450,630)$
	A1	For any two correct angles from 45°, 135°, 225° or 315°
	A1	For all four angles from $\cos x$ or $\cos 2x$
	For extra angles within range, penalise the last A mark only. For example; If they have [coming from a correct method] *x* = 45°, 67.5°, 135°, 225°, 315° score M1A1A0 If they have *x* = 45°, 67.5°, 135° then award M1A1A0	

Question number	Scheme	Marks
11 a	When $x=0 \quad y=3$	B1
	When $y=0 \quad 0=4-\mathrm{e}^{2x} \Rightarrow \ln 4 = 2x \Rightarrow x=\ln 2$	M1 A1 (3)
b	$\frac{\mathrm{d}y}{\mathrm{d}x}=-2\mathrm{e}^{2x}$	M1
	When $x=\ln 2 \quad \frac{\mathrm{d}y}{\mathrm{d}x}=-8$	M1
	$y-0=\frac{1}{8}(x-\ln 2) \Rightarrow y=\frac{1}{8}x-\frac{1}{8}\ln 2$	M1 A1 (4)
c	$\int_0^{\ln 2}\left(4-\mathrm{e}^{2x}\right)\mathrm{d}x=\left[4x-\frac{1}{2}\mathrm{e}^{2x}\right]_0^{\ln 2}=\left(4\ln 2-\frac{1}{2}\mathrm{e}^{2\ln 2}\right)-\left(0-\frac{1}{2}\mathrm{e}^{0}\right)$	M1M1M1
	$\frac{1}{2}(\ln 2)\left(\frac{1}{8}\ln 2\right)=\frac{1}{16}(\ln 2)^2$	M1M1
	$\frac{1}{8}\int_0^{\ln 2}\lvert x-\ln 2\rvert\,\mathrm{d}x=\frac{1}{8}\left\lvert\frac{x^2}{2}-x\ln 2\right\rvert_0^{\ln 2}=\left\lvert\frac{(\ln 2)^2}{16}-\frac{(\ln 2)^2}{8}\right\rvert=\frac{(\ln 2)^2}{16}$	[M1M1]
	$4\ln 2-\frac{3}{2}+\frac{1}{16}(\ln 2)^2=1.3$	M1 A1 (7)
		Total 14 marks

USEFUL SKETCH



Part	**Mark**	**Notes**

(a)(i)	B1	For $y = 3$
(ii)	M1	For $\ln 4 = 2x$ or $\sqrt{4} = \sqrt{e^{2x}}$ seen explicitly
	A1	For $x = \ln 2$
(b)	M1	For differentiating the given expression. This must be correct for this mark. $\frac{dy}{dx} = -2e^{2x}$
	M1	For substituting ln2 into their $\frac{dy}{dx} = -2e^{2\ln 2} = [-8]$
	M1	For a correct method for finding the equation of a straight line using their numerical perpendicular gradient and $y = 0$ and $x = \ln 2$ Award when they substitute into a correct formula, or if they use $y = mx + c$ award when c is obtained. (accept a decimal value for c for this mark awrt $c = -0.087$)
	A1	For $y = \frac{1}{8}x - \frac{1}{8}\ln 2$ or $y = \frac{x}{8} - \frac{\ln 2}{8}$ in exact form only.
(c)	**Area under curve**	
	M1	For a correct statement for the area under the curve with correct limits. Accept the limits either way around. Ignore poor notation. This mark can be implied by later correct work. $\int_0^{\ln 2} \left(4 - e^{2x}\right) dx$
	M1	For a minimally acceptable attempt to integrate as follows. $\int_0^{\ln 2} \left(4 - e^{2x}\right) dx = \left[4x \pm \frac{e^{2x}}{2}\right]$
	M1	For substitution of both limits into their integral of the curve. $A = \left(4\ln 2 - \frac{1}{2}e^{2\ln 2}\right) - \left(0 - \frac{1}{2}e^{0}\right) = \left[4\ln 2 - \frac{3}{2}\right] = [1.27258....]$
	Area of the triangle	
	M1	**Method 1** For a statement of the area. $A = \frac{1}{2}(\ln 2)\left('\frac{1}{8}\ln 2'\right)$ **Method 2** For a statement of the area, with limits either way around. $A = \frac{1}{8}\int_0^{\ln 2} (x - \ln 2)\, dx$ ft their equation of the line.
	M1	For a correct method to evaluate the area of the triangle. **Method 1** $A = \frac{1}{2}(\ln 2)\left('\frac{1}{8}\ln 2'\right) = \frac{1}{16}(\ln 2)^2 = [0.03002.....]$ **Method 2** The integration and substitution must be correct for this mark $A = \frac{1}{8}\int_0^{\ln 2} \lvert x - \ln 2\rvert\, dx = \frac{1}{8}\left\lvert \frac{x^2}{2} - x\ln 2 \right\rvert_0^{\ln 2} = \left\lvert \frac{(\ln 2)^2}{16} - \frac{(\ln 2)^2}{8} \right\rvert = \frac{(\ln 2)^2}{16}$

	Combined area	
	M1	For the total final area using their values $4\ln 2 - \frac{3}{2} + \frac{1}{16}(\ln 2)^2$ **This is an A mark in Epen.**
	A1	For awrt 1.3 Also accept the exact Area = $4\ln 2 - \frac{3}{2} + \frac{1}{16}(\ln 2)^2$

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