Mark Scheme (Results)

Summer 2016

Pearson Edexcel International GCSE in Further Pure Mathematics Paper 2 (4PM0/02)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers may score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q) \text{ where } |pq| = |c|$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a|$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to x=…..

3. Completing the square:

Solving $x^2 + bx + c = \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$ where $q \neq 0$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

Use of a formula:

Generally, the method mark is gained by

either quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1 (a)	$\cos\theta° = \dfrac{8^2 + 9^2 - 10^2}{2 \times 8 \times 9}$	M1A1
	$\theta° = 71.79... = 71.8°$	A1 cao (3)
(b)	Area $= \dfrac{1}{2}ab\sin C = \dfrac{1}{2} \times 8 \times 9 \sin 71.79...$	M1
	$= 34.19... = 34.2$ (cm^2) (Use of 71.8 also gives 34.2)	A1cao (2) [5]
(a)M1	Cosine rule for any angle of the triangle; can be in either form but formula must be correct	
A1	Correct numbers in the cosine rule. Must be the correct angle (ie largest)	
A1cao	Identify $\theta = 71.8°$ Must be to nearest 0.1°	
ALT:	Find at least 2 angles by cosine and possibly sine rule. (can be any 2 of the angles) M1A1	
	$\theta = 71.8°$ Must be to nearest 0.1° A1	
(b)		
M1	Any complete method to find the area of the triangle (use any angle found in (a) with the sides enclosing it)	
A1cao	34.2 Must be to 3sf unless rounding already penalised in (a)	

Question Number	Scheme	Marks
2 (a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}, = (3\mathbf{i}+9\mathbf{j}) - (6\mathbf{i}+5\mathbf{j}) = -3\mathbf{i}+4\mathbf{j}$	M1,A1cao (2)
(b)	$\dfrac{\lambda}{12} = \dfrac{4}{(-)3}, \quad \lambda = -16$	M1,A1cao (2)
ALT:	$\overrightarrow{PQ} = \mu\overrightarrow{AB} \quad 12\mathbf{i}+\lambda\mathbf{j} = \mu(-3\mathbf{i}+4\mathbf{j})$ M1 (Their $\overrightarrow{AB}$) Allow $\mu = \dfrac{12\mathbf{i}+\lambda\mathbf{j}}{-3\mathbf{i}+4\mathbf{j}}$ $\mu = -4 \quad \lambda = -16$ A1	
(c)	$\lvert\overrightarrow{AB}\rvert = \sqrt{(3^2+4^2)} = 5$ or $\lvert\overrightarrow{PQ}\rvert = 20$	M1
	$= \pm\dfrac{1}{5}(3\mathbf{i}-4\mathbf{j})$ oe	A1 (2)
		[6]
(a)M1	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or $\overrightarrow{OB} + \overrightarrow{AO}$ or use a diagram. Column vectors allowed for the M mark.	
A1cao	$-3\mathbf{i}+4\mathbf{j}$ or $4\mathbf{j}-3\mathbf{i}$ or $\begin{pmatrix}-3\mathbf{i}\\4\mathbf{j}\end{pmatrix}$ but **i**, **j** must be included	
(b)M1	Finding and equating the gradients of the two lines. Fractions can be either way up as long as consistent and attempting to solve for λ There may be sign errors in the equation. Or compare the components. **NB:** Using $\overrightarrow{PQ} = \overrightarrow{AB}$ scores M0 unless a fresh start is made.	
A1cao	$\lambda = -16$	
(c)M1	Use Pythagoras with a + sign to obtain the length of their *AB* or their *PQ*	
A1	A correct unit vector in either direction and any equivalent form inc column vector	

Question Number	Scheme	Marks
3 (a)	$\dfrac{(5x+3)}{(11x-3)}=\dfrac{(3x-3)}{(5x+3)}$ or $(5x+3)^2=(3x-3)(11x-3)$	M1A1
	$25x^2+30x+9=33x^2-42x+9$	
	$8x^2-72x(=0)$ $x=0,\ x=9$	dM1A1 (4)
	Spec case: Give M1A0M0A0 (ie B1) if x = 0 seen w/o working	
(b)	$x=0$ $r=\dfrac{3}{-3}=-1$	B1
	$x=9$ $r=\dfrac{48}{96}=\dfrac{1}{2}$	M1A1cso (3)
(c)	$x=9$ $a=96$	
	$S_\infty=\dfrac{96}{1-\frac{1}{2}},=192$	M1Aft, A1cao (3)
		[10]
(a)M1	Form an equation connecting the three given terms, must either be equating fractions or multiplying in pairs.	
A1	Correct equation, fractions can be either way up	
dM1	Solve the resulting quadratic to x = ...	
A1	**Both c**orrect values of x obtained	
(b)		
B1	$r=-1$ seen	
M1	Use a non-zero value of x obtained in (a) and obtain the corresponding of r. Must use the same value of x in both substitutions.	
A1cso	$r=\dfrac{1}{2}$	
(c)M1	Use the formula for the sum to infinity of a convergent geometric series with $\lvert r\rvert<1$ and a value of a found using the corresponding value of x.	
	Acceptable formulae $S_\infty=\dfrac{a}{1-r},=\dfrac{a(1-r^\infty)}{1-r},=\dfrac{a(r^\infty-1)}{r-1}$	
A1ft	"Correct" numbers in the formula, ft their x and r **and** $r^\infty=0$	
A1cao	192	

Question Number	Scheme	Marks
4	$\frac{dy}{dx} = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$	M1A1A1 [3]
M1	Differentiate wrt x. Two terms either added or subtracted. Terms to be one of each of $pe^{2x}\cos 3x$ and $qe^{2x}\sin 3x$ where p and q are integers.	
A1	Either term correct	
A1	The other term correct	
NB	If the product rule is quoted and brackets omitted on application eg $2e^{2x}\cos 3x + e^{2x} - 3\sin 3x$ allow for "invisible brackets" and award M1A1A0. If final statement fully correct award M1A1A1	

Question Number	Scheme	Marks
5(a)	$V = x \times 4x \times h = 772$	
	$h = \dfrac{193}{x^2} = \dfrac{772}{4x^2}$	B1
	$A = 2\times 4x^2 + 2xh + 2\times 4xh = 8x^2 + 10xh$	
	$A = 8x^2 + 10x \times \dfrac{193}{x^2} = 8x^2 + \dfrac{1930}{x}$ *	M1A1cso (3)
(b)	$(A =)8x^2 + 1930x^{-1}$	
	$\left(\dfrac{dA}{dx} = \right)16x - 1930x^{-2}$	M1
	$\left(\dfrac{dA}{dx} = 0 \Rightarrow\right)16x = \dfrac{1930}{x^2}$	dM1
	$x^3 = \dfrac{1930}{16}$ $x = 4.9409... = 4.94$	A1
	$\left(\dfrac{d^2A}{dx^2} = \right)16 + 3860x^{-3}$	M1
	$x = 4.94 \Rightarrow \dfrac{d^2A}{dx^2} > 0$ $\therefore$ minimum	A1ft (5)
(c)	$A_{\text{min}} = 8\times 4.940...^2 + \dfrac{1930}{4.940...} = 585.9..., = 586$	M1,A1cao (2)
		[10]

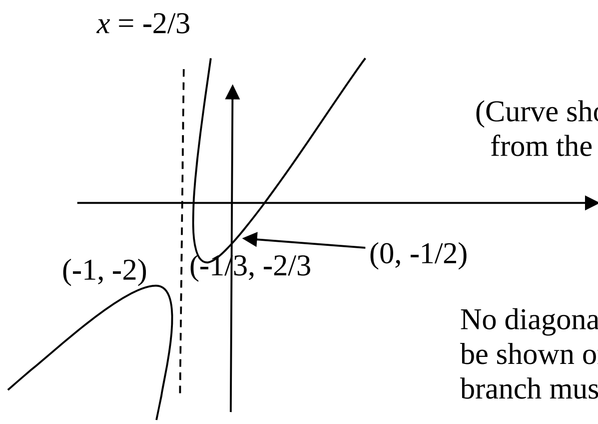
(a)B1	$h = \dfrac{193}{x^2}$ or $\dfrac{772}{4x^2}$ or $xh = \dfrac{193}{x}$ oe Seen explicitly or used in the expression for A
M1	Form an expression for A in terms of x and h which must be dimensionally correct and replace h with a function of x
A1cso	Obtain the given expression for A. No errors seen. Must start A = ... or area = ...
(b)	
M1	Differentiate the GIVEN expression for A
dM1	Equate their derivative to 0. Dependent on the previous M mark.
A1	$x = 4.94$ Must be 3 sf
	Special Case: 4.94 with no working (calculator solution) scores M1M1A1
M1	Attempt the second derivative - must have 2 terms.
A1ft	Deduce that their value of x gives a minimum, follow through their x. No need to evaluate the derivative provided the value of x is positive and the derivative is algebraically correct. Must have a conclusion.
ALTs	For last 2 marks: Look at signs of $\dfrac{dA}{dx}$ either side of $x = 4.94$ **and** calculate the values of $\dfrac{dA}{dx}$ (M1) All correct with conclusion (A1) or refer to the graph - sketch must be shown.
(c)	
M1	Use their value of x in the given expression for A and complete to A = ...
A1cao	586 Must be 3sf unless rounding already penalised in (b)

Question Number	Scheme	Marks
6 (a)	$5x+4=x^2+2x-6$	M1
	$x^2-3x-10(=0)$	A1
	$(x-5)(x+2)(=0)$	
	$x=5\;\; y=29\;;\quad x=-2\;\; y=-6$	M1A1A1 (5)
(b)	$\int_{-2}^{5}\left((5x+4)-\left(x^2+2x-6\right)\right)\mathrm{d}x$ (either way round)	M1
	$\int_{-2}^{5}\left(-x^2+3x+10\right)\mathrm{d}x$	
	$\left[-\frac{1}{3}x^3+\frac{3}{2}x^2+10x\right]_{-2}^{5}$ (Correct integration of a function, either way round or correct integration of two sep functions)	M1A1
	$=\left(-\frac{125}{3}+\frac{75}{2}+50\right)-\left(\frac{8}{3}+6-20\right)$	dM1
	$=57\frac{1}{6},\ \frac{343}{6}$ **must** be positive	A1cao (5)
		[10]

(a)	
M1	Eliminate y or x between the two equations to obtain an equation in a single variable
A1	Correct 3 term quadratic
M1	Solve their 3TQ to x = ... or y = ... Calculator solutions must have $x=-2$ and 5 or $y=-6$ and 29 ie both solutions for their variable.
A1	Either pair of coordinates correct
A1	Second pair correct. Coordinate brackets not needed but some indication of pairing is needed
NB	Table of values methods score 0/5
(b)	
M1	For the integral of "line - curve", either way round. Ignore any limits shown. This mark can be given later if two separate integrals are used - give when the difference of the two integrals is shown.
M1	Integration of the function, either way round or correct integration of two separate functions
A1	Correct integration. Ignore limits for these two marks.
dM1	Substitute their limits (ie their values found in (a)) in the integral of the single function or in both integrals. Both the above M marks must be earned.
A1cao	Area $=57\frac{1}{6}$ oe must be positive.
NB	If only the line or the curve is integrated score is 0/5

Question Number	Scheme	Marks
7(a)	$\frac{dv}{dt} = 6t - 4$	M1
	$t = 2$ accel $= 8$ (m/s^2)	A1 (2)
(b)	v is min when $\frac{dv}{dt} = 0$ ie when $t = \frac{2}{3}$	M1
	$v_{\min} = 3\times\left(\frac{2}{3}\right)^2 - 4\times\frac{2}{3} + 7 = 5\frac{2}{3}$ (m/s) (Accept 5.67, $\frac{17}{3}$)	M1A1 (3)
(c)	$V = 7$	B1 (1)
(d)	$3t^2 - 4t + 7 = 7$	
	$t(3t-4) = 0$ $(t = 0)$ $t = \frac{4}{3}$	B1
	$AB = \int_0^{\frac{4}{3}} (3t^2 - 4t + 7)\,dt$	M1
	$= \left[t^3 - 2t^2 + 7t\right]_0^{\frac{4}{3}}$	M1A1
	$= \left(\frac{4}{3}\right)^3 - 2\times\left(\frac{4}{3}\right)^2 + 7\times\frac{4}{3}$ (-0)	dM1
	$= 8\frac{4}{27}$ (m), $\frac{220}{27}$ (Accept 8.15 or better)	A1cao,cso (6)
		[12]

(a)	
M1	Differentiate given expression for v wrt t. Power must decrease on at least one term
A1	Substitute $t = 2$ and obtain $\text{accel} = 8$ (m/s^2)
	Correct answer with no working shown Award both marks
(b)	
M1	Set their $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ and solve to t = ... or deduce the nec value of t from work in (a)
M1	Substitute their value of t in the GIVEN expression for v
A1	$5\frac{2}{3}$ or $\frac{17}{3}$ or 5.67 Decimal to be 3 sf or better
	Correct answer with no working Award 3/3
ALT	Complete the square on v $v = 3\left(t - \frac{2}{3}\right)^2 + 7 - ...$ M1
	Identify the constant "$7 - \frac{4}{3}$" as the min or take $t = \frac{2}{3}$ from bracket and substitute M1
	Correct answer A1
(c)	
B1	$(V =) 7$ Need not say $V =$
(d)	
B1	Equate the expression for v to 7 and solve to obtain t = 4/3 No need to show t = 0
M1	Form the required integral with lower limit = 0 and their value of t as the upper limit Do NOT give this mark until limits are seen.
M1	Attempt the integration. Power to increase on at least one term.
A1	Correct integration
dM1	Substitute their upper limit in a changed function. Depends on the first M mark but not the second. Sub of lower limit need not be seen (it gives 0)
A1cao cso	$8\frac{4}{27}$ or $\frac{220}{27}$ Decimal to be 3 sf or better
ALT:	By Indefinite integration: B1 As above; M1 Do NOT award until t = 0 is used to find the constant of integration M1A1 for the integration constant can be omitted dM1A1 as above

Question Number	Scheme	Marks
8(a)	$x=-\frac{2}{3}$	B1 (1)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6x(3x+2)-3(3x^2-1)}{(3x+2)^2}$	M1A1A1
	$18x^2+12x-9x^2+3=0$ oe	M1
	$3x^2+4x+1=0$	A1
	$(3x+1)(x+1)=0 \qquad x=-\frac{1}{3} \quad x=-1$	M1A1
	$\left(-\frac{1}{3},-\frac{2}{3}\right) \quad (-1,-2)$	A1 (8)
(c)	A is $\left(0,-\frac{1}{2}\right)$	B1 (1)
(d)	 (Curve should not bend away from the asymptote.) No diagonal asymptote need be shown or implied but each branch must have a turning point.	B1 two branches with **turning points** B1 Asymptote parallel to y-axis B1 Required coords (3)
(e)	grad at $A=\frac{0-3(-1)}{2^2}=\frac{3}{4}$ grad normal $-\frac{4}{3}$	B1
	$y+\frac{1}{2}=-\frac{4}{3}x$ oe	M1A1 (3)
		[16]

8(a)	
B1	$x=-\frac{2}{3}$ oe eg $3x=-2,\ 3x+2=0$ Must be an equation
(b)	
M1	Attempt to differentiate using the quotient or product rule. For quotient rule, the numerator must be the difference of 2 terms and the denominator must be $(3x+2)^2$ For the product rule the difference of 2 terms is required and both terms must contain $(3x+2)^{-k}$, where $k=1$ or 2
A1	For quotient rule, either term correct apart from sign For product rule, either term correct
A1	Completely correct derivative.
M1	Equate their numerator to 0. (For product rule use, equate their whole derivative to 0)
A1	Simplify to the correct 3 term quadratic. Terms can be in any order.
M1	Attempt the solution of their 3 term quadratic
A1	**Two** correct values for x
A1	Corresponding correct values for y. No need to write in coordinate brackets.
(c)	
B1	$\left(0,-\frac{1}{2}\right)$ or $x=0, y=-\frac{1}{2}$
(d)	
B1	Two branches with turning points. One to be in all 4 quadrants, the other in the third quadrant only
B1	Vertical asymptote drawn and labelled either with its equation or by the point where it crosses the *x*-axis. At least one branch of the curve must be asymptotic to the line and neither branch should cross it.
B1	Show the required coordinates on their sketch beside their turning points or indicated by arrow(s).
(e)	
B1	Gradient of the normal seen explicitly or used.
M1	Any **complete** method for the equation of a line using their gradient of the normal at *A* and their coordinates of *A*
A1	Correct equation, any equivalent form.

Question Number	Scheme	Marks
9		
(a)	$\cos(\theta+\theta)=\cos\theta\cos\theta-\sin\theta\sin\theta$	M1
	$\cos 2\theta=\cos^2\theta-\left(1-\cos^2\theta\right)$	M1
	$\cos 2\theta=2\cos^2\theta-1$ *	A1cso (3)
(b)	$\sin 2\theta=2\sin\theta\cos\theta$	B1 (1)
(c)	$\cos 3\theta=\cos 2\theta\cos\theta-\sin 2\theta\sin\theta$	M1
	$=\left(2\cos^2\theta-1\right)\cos\theta-2\sin\theta\cos\theta\sin\theta$	M1
	$=2\cos^3\theta-\cos\theta-2\left(1-\cos^2\theta\right)\cos\theta$	M1
	$=4\cos^3\theta-3\cos\theta$ *	A1cso (4)
(d)	$1=8\cos^3\theta-6\cos\theta=2\cos 3\theta$	
	$\cos 3\theta=\frac{1}{2}$	M1
	$3\theta=\frac{\pi}{3},\ \frac{5\pi}{3},\ \frac{7\pi}{3}$	M1
	$\theta=\frac{\pi}{9},\ \frac{5\pi}{9},\ \frac{7\pi}{9}$	A1A1 (4)
(e)	$\int\left(8\cos^3\theta+4\sin\theta\right)\mathrm{d}\theta=\int\left(2\cos 3\theta+6\cos\theta+4\sin\theta\right)\mathrm{d}x$	M1
	$=\frac{2}{3}\sin 3\theta+6\sin\theta-4\cos\theta\ (+c)$	A1
(ii)	$=\frac{2}{3}\sin\pi+6\sin\frac{\pi}{3}-4\cos\frac{\pi}{3}-\left(-4\cos 0\right)$	dM1
	$=6\times\frac{\sqrt{3}}{2}-2+4\ =3\sqrt{3}+2$	A1cao cso (4)
		[16]

9	If c, s used for cos and sin allow for all marks except final A marks in each section. For these marks the candidate must return to cos, sin as appropriate.
(a)	
M1	Replace A and B with θ in $\cos(A+B)=\cos A\cos B-\sin A\sin B$
M1	Use $\sin^2\theta=1-\cos^2\theta$ to eliminate $\sin^2\theta$
A1cso	Obtain the **given** identity with no errors seen
(b)	
B1	$\sin 2\theta=2\sin\theta\cos\theta$ Must be simplified
(c)	
M1	Use $\cos(A+B)=\cos A\cos B-\sin A\sin B$ with $A=2\theta,\ B=\theta$ to eliminate 3θ
M1	Use $\cos 2\theta=2\cos^2\theta-1$ **and** $\sin 2\theta=2\sin\theta\cos\theta$ to obtain an expression with powers of $\sin\theta$ and $\cos\theta$
M1	Use $\sin^2\theta=1-\cos^2\theta$ to eliminate $\sin^2\theta$ leaving powers of cos only
A1cso	Obtain the **given identity** with no errors seen
(d)	
M1	Use the identity given in (c) to change the equation to the form $\cos 3\theta=k$ where $-1<k<1$
M1	Obtain 1 value in range $0\leqslant 3\theta<3\pi$ in terms of π for 3θ
A1	Any 2 correct values for θ
A1	Third correct value for θ Ignore any answers outside the range, deduct one A mark if extras within the range are included.
ALT:	Can work in degrees for the M marks; A marks only available if answers changed to radians without loss of accuracy.
(e)	
(i) M1	Change the given integrand to one which can be integrated, either by using the identity in (c) or by obtaining $\int\left(8\cos^3\theta+4\sin\theta\right)\mathrm{d}\theta=\int\left(8\left(1-\sin^2\theta\right)\cos\theta+4\sin\theta\right)\mathrm{d}x$
A1	Correct integration $\frac{2}{3}\sin 3\theta+6\sin\theta-4\cos\theta\ (+c)$ or $8\sin\theta-\frac{8}{3}\sin^3\theta-4\cos\theta\ (+c)$ Constant of integration may be missing.
(ii) dM1	Substitute both limits into their answer from (i)- evidence needed of substitution of 0 Substitution of upper limit followed by - 0 qualifies
A1cao cso	$3\sqrt{3}+2$ oe two terms only.
	Watch for: $\int\left(8\cos^3\theta+4\sin\theta\right)\mathrm{d}\theta=\left[8\sin^3\theta-4\cos\theta\right]_0^{\frac{\pi}{3}}$ $=3\sqrt{3}-2-(-4)$ $=$ correct answer!! But from completely INCORRECT working.

Question Number	Scheme	Marks
10(a)	$V=\frac{1}{3}\pi hr^2=\frac{1}{3}\pi h\times\left(h\tan 30\right)^2 \quad \left(=\frac{1}{3}\pi h^3\times\left(\frac{1}{\sqrt{3}}\right)^2=\frac{1}{9}\pi h^3\right)$	B1
	$V=0.4t$	B1
	$0.4t=\frac{2}{5}t=\frac{1}{9}\pi h^3$	M1
	$h^3=\frac{18t}{5\pi}$ *	A1cso (4)
(b)	Area of top $=\pi\left(h\tan 30\right)^2=\frac{1}{3}\pi h^2$	B1
	$\frac{\mathrm{d}A}{\mathrm{d}h}=\frac{2}{3}\pi h$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$	M1
	$h^3=\frac{18t}{5\pi}$	
	$3h^2=\frac{18}{5\pi}\frac{\mathrm{d}t}{\mathrm{d}h}$	M1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{6}{5\pi h^2}$	A1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{2}{3}\pi h\times\frac{6}{5\pi h^2}=\frac{4}{5h}$ *	A1cao (6)
(c)	$t=10 \quad h=\sqrt[3]{\frac{180}{5\pi}} \qquad \frac{\mathrm{d}A}{\mathrm{d}t}=\frac{4}{5h}=0.355\ \text{cm}^2/\text{s}$	M1A1cao (2)
		[12]

10(a)B1	$V=\frac{1}{3}\pi h\times\left(h\tan 30\right)^2$ $\left(\text{or } V=\frac{1}{9}\pi h^3\right)$ (ie replace r)
B1	$V=0.4t$
M1	Equating their 2 expressions for V to obtain an equation without r
A1cso	Re-arrange to $h^3=\frac{18t}{5\pi}$ with no errors seen
(b)	
B1	Area of top $=\frac{1}{3}\pi h^2$
M1	Differentiate their expression for the area of the top wrt h
M1	Chain rule connecting $\frac{\mathrm{d}A}{\mathrm{d}t}$, $\frac{\mathrm{d}A}{\mathrm{d}h}$ and $\frac{\mathrm{d}h}{\mathrm{d}t}$, any equivalent form or a useful chain rule with more derivatives eg $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ see alt solution below
M1	Differentiate the **given** expression from (a) wrt h or t
A1	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{6}{5\pi h^2}$ or $\frac{\mathrm{d}t}{\mathrm{d}h}=\frac{5\pi h^2}{6}$ or any equivalent expression in terms of t.
A1cao	Substitute for $\frac{\mathrm{d}A}{\mathrm{d}h}$ and $\frac{\mathrm{d}h}{\mathrm{d}t}$ in the chain rule to obtain the **given** expression for $\frac{\mathrm{d}A}{\mathrm{d}t}$ No errors seen.
ALTs:	
1	Using $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ B1 $\frac{\mathrm{d}A}{\mathrm{d}r}=2\pi r$ M1 Find $\frac{\mathrm{d}r}{\mathrm{d}h}=\frac{1}{\sqrt{3}}$ M1 Chain rule $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ M1A1A1 As main scheme
2	Using $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ B1M1 As main scheme M1 Chain rule $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ M1 Attempt $\frac{\mathrm{d}V}{\mathrm{d}h}$ using their expression for V in terms of h found in (a) A1 $\frac{\mathrm{d}V}{\mathrm{d}t}=0.4$ A1 Complete to required result.
3	Using $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ B1 $\frac{\mathrm{d}A}{\mathrm{d}r}=2\pi r$ M1 $t=\frac{5}{6}\pi r^2h$ (Obtained from $0.4t=\frac{1}{3}\pi r^2h$ (in (a)) M1 $\frac{\mathrm{d}t}{\mathrm{d}r}=\frac{5\sqrt{3}}{2}\pi r^2$ M1 $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}\left(=2\pi r\times\frac{2}{5\sqrt{3}\pi r}=\frac{4}{5\sqrt{3}r}\right)$ A1 Use $h=\sqrt{3}r$ in their $\frac{\mathrm{d}A}{\mathrm{d}t}$ A1 Correct final result, no errors seen

(c)	
M1	Use $t = 10$ to obtain the corresponding value of h $h = \left(\sqrt[3]{\dfrac{180}{5\pi}} \text{ or } 2.2545...\right)$ and substitute their value of h in the expression from (b) to obtain $\dfrac{dA}{dt} = ...$
A1cao	$\dfrac{dA}{dt} = 0.355\,(\text{cm}^2/\text{s})$ **Must** be 3sf.

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