

Mark Scheme (Provisional)

Summer 2021

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 02

### **Edexcel and BTEC Qualifications**

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at <a href="https://www.edexcel.com">www.edexcel.com</a> or <a href="https://www.edexcel.com">www.btec.co.uk</a>. Alternatively, you can get in touch with us using the details on our contact us page at <a href="https://www.edexcel.com/contactus">www.edexcel.com/contactus</a>.

# Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: <a href="https://www.pearson.com/uk">www.pearson.com/uk</a>

Summer 2021
Question Paper Log Number P66025A
Publications Code 4PM1\_02\_2106\_MS
All the material in this publication is copyright
© Pearson Education Ltd 2021

## **General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

# Types of mark

- o M marks: method marks
- o A marks: accuracy marks
- o B marks: unconditional accuracy marks (independent of M marks)

#### Abbreviations

- o cao correct answer only
- o ft follow through
- o isw ignore subsequent working
- o SC special case
- oe or equivalent (and appropriate)
- o dep dependent
- o indep independent
- o awrt answer which rounds to
- o eeoo each error or omission

#### No working

If no working is shown then correct answers normally score full marks
If no working is shown then incorrect (even though nearly correct) answers score
no marks.

### With working

If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.

If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.

If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

### • Ignoring subsequent work

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

# • Parts of questions

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

# **General Principles for Further Pure Mathematics Marking**

(but note that specific mark schemes may sometimes override these general principles)

## Method mark for solving a 3 term quadratic equation:

### 1. Factorisation:

$$(x^2+bx+c)=(x+p)(x+q)$$
, where  $|pq|=|c|$  leading to  $x=...$   
 $(ax^2+bx+c)=(mx+p)(nx+q)$  where  $|pq|=|c|$  and  $|mn|=|a|$  leading to  $x=...$ 

## 2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to x = ...

3. Completing the square:

$$x^{2} + bx + c = 0$$
:  $(x \pm \frac{b}{2})^{2} \pm q \pm c = 0$ ,  $q \ne 0$  leading to  $x = ...$ 

# Method marks for differentiation and integration:

### 1. <u>Differentiation</u>

Power of at least one term decreased by 1.  $(x^n \rightarrow x^{n-1})$ 

### 2. Integration:

Power of at least one term increased by 1.  $(x^n \rightarrow x^{n+1})$ 

#### Use of a formula:

Generally, the method mark is gained by **either** 

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is <u>not</u> quoted, the method mark can be gained by implication from the substitution of <u>correct</u> values and then proceeding to a solution.

# **Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

#### **Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

# Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

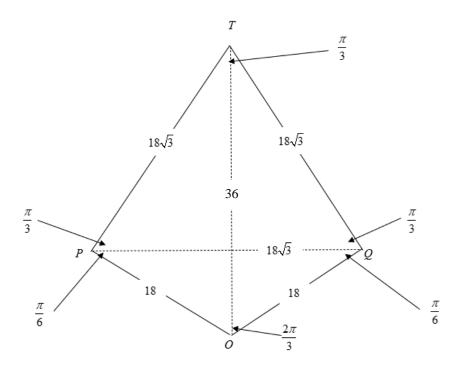
Paper 1			
Question	Scheme	Marks	
number			
1 (a)	3x < 12	M1	
	x < 4	A1	
		[2]	
(b)	(2x+1)(x-3) > 0	M1	
	$(2x+1)(x-3) > 0$ Critical values are $x = -\frac{1}{2}$ and $x = 3$	M1	
	$x < -\frac{1}{2}$ $x > 3$	A1	
		[3]	
(c)	$x < -\frac{1}{2}  3 < x < 4$	B1ft	
	2	[1]	
		Total 6 marks	

Part	Mark	Guidance				
(a)	M1	Attempts to solve the inequality to achieve $3x < 12$				
	1411	Allow $3x < a$ where a is an integer				
	<b>A1</b>	For $x < 4$				
<b>(b)</b>		Attempts to solve the inequality by any method to find critical values				
		See General Guidance for acceptable methods.				
		If a calculator is used, the solution must be fully correct for this mark.				
	M1	Allow = or > for this mark or even no sign at all provided it is clear they are solving a quadratic.				
		$(2x+1)(x-3) > 0 \Rightarrow x =, \left(x = -\frac{1}{2}, 3\right)$				
	M1	For forming a correct inequality, which must be an open interval, following through their <b>two</b> critical values which must have come from the solution of a 3TQ. $x < -\frac{1}{2}  x > 3$ Accept any correct notation. E.g., $x < -\frac{1}{2}$ or $x > 3$ Or $\left\{x: x < -\frac{1}{2}\right\} \cup \left\{x: x > 3\right\}$				
		Condone $x < -\frac{1}{2}$ and $x > 3$ for this mark only				
	<b>A1</b>	For the correct inequality with the correct critical values using any acceptable notation.  Eg, $x < -\frac{1}{2}$ $x > 3$				
(c)		Eg, $x < -\frac{1}{2}$ $x > 3$ For $x < -\frac{1}{2}$ $3 < x < 4$				
	B1ft	ft their answers from parts (a) and (b) provided (b) is of the form $x < p$ and/or $x > q$				

Question number	Scheme	Marks
2 (a)	$2 - \frac{1}{25} (x^2 - 20x)$ $2 \mp \frac{1}{25} [(x \pm 10)^2 - 100]$	
	$2 \mp \frac{1}{25} \Big[ (x \pm 10)^2 - 100 \Big]$	M1
	$6 - \frac{1}{25}(x - 10)^2 \qquad \text{So } A = 6  B = \frac{1}{25}  C = -10$	A1 A1 A1 (4)
ALT	$A - Bx^2 - 2BCx - BC^2 = 2 + \frac{4}{5}x - \frac{1}{25}x^2$	{M1}
	$B = \frac{1}{25}$	{A1}
	$-\frac{2}{25}C = \frac{4}{5} \qquad C = -10$ $-\frac{1}{25}(100) + A = 2 \qquad A = 6$	{A1}
	$-\frac{1}{25}(100) + A = 2 \qquad A = 6$	{A1} (4)
(b)(i)	6	B1 ft
(b)(ii)	10	B1 ft
	Tota	(2) l 6 marks

Part	Mark	Guidance		
(a)	<b>M1</b>	For a complete method to complete the square to achieve as a minimum		
		$2 \mp \frac{1}{25} (x \pm 10)^2 - p$ or $\mp \frac{1}{25} [(x \pm 10)^2 - q - 50]$		
		where $p$ and $q$ are constants		
	A1	For one correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated		
		explicitly or embedded		
	<b>A1</b>	For two correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated		
		explicitly or embedded		
	A1	Fully correct $A = 6$ $B = \frac{1}{25}$ and $C = -10$ <b>OR</b> $6 - \frac{1}{25}(x - 10)^2$ oe		
		equates coefficients		
	M1	For an attempt to expand $A - B(x+C)^2$ <b>AND</b> equate coefficients to the		
		given $f(x) \Rightarrow A - Bx^2 - 2BCx - BC^2 = 2 + \frac{4}{5}x - \frac{1}{25}x^2$		
		Allow $A \pm Bx^2 \pm 2BCx \pm BC^2$ for the expansion of $A - B(x + C)^2$		
		There must be an attempt to equate at least one coefficient.		
		$-B = -\frac{1}{25} \Longrightarrow B = \dots$		
		$-2BC = \frac{4}{5} \Rightarrow C = \dots$		
		$A - BC^2 = 2 \Rightarrow A = \dots$		
	<b>A1</b>	For one correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated		
		explicitly or embedded		
	<b>A1</b>	For two correct from $A = 6$ $B = \frac{1}{25}$ or $C = -10$ whether stated		
		explicitly or embedded		
	A1	Fully correct $A = 6$ $B = \frac{1}{25}$ and $C = -10$ <b>OR</b> $6 - \frac{1}{25}(x - 10)^2$ oe		
(b)(i)	B1ft	For the value of 6 or ft their <i>A</i>		
(ii)	B1ft	For the value of 10 or ft their <i>C</i>		

Question number	Scheme	Marks
3 (a)	$PQ = 18 \times \frac{2}{3}\pi = 12\pi$	M1 A1 (2)
(b)(i)	$\alpha = \frac{1}{3}\pi$	B1
(6)(11)	$PT = 18\tan\frac{\pi}{3} = 18\sqrt{3}$	M1 A1
	Area of $OPTQ = 2 \times \frac{1}{2} \times 18 \times 18\sqrt{3}$	M1
	Area of Sector $OPQ = \frac{1}{2} \times 18^2 \times \frac{2\pi}{3}$	M1
	Shaded Area = $2 \times \frac{1}{2} \times 18 \times 18\sqrt{3} - \frac{1}{2} \times 18^2 \times \frac{2\pi}{3} = 222 \text{ cm}^2$	M1 A1 (7)
	Tota	l 9 marks



Area of triangle  $OPQ = 81\sqrt{3}$  or 140.29... cm<sup>2</sup>

Area of triangle  $PQT = 243\sqrt{3}$  or 420.88... cm<sup>2</sup>

Area of quadrilateral  $OTPQ = 324\sqrt{3}$  or 561.18... cm<sup>2</sup>

Part	Mark	Guidance
(a)	M1	Uses the <b>correct</b> formula for the length of arc to give $PQ = 18 \times \frac{2}{3} \pi = \dots$
	<b>A1</b>	For $PQ = 12\pi$

	AIT	ATT			
	ALT – works in degrees (but the angle must be correct at 120°)  Uses the correct formula for length of arc to give				
	3.71	Uses the <b>correct</b> formula for length of arc to give			
	M1	$PQ = \frac{120}{360} \times 2\pi \times 18 = \dots$			
	A 4				
<b>a</b> >	A1	For $PQ = 12\pi$			
(b)	(i) B1	For stating $\alpha = \frac{\pi}{3}$ (Please check the diagram as it may written on there)			
	(ii) Me correct.	ethod 1 - Allow use of degrees throughout provided the angles are			
		$\left(\angle POQ = 120^{\circ}, \ \angle PTQ = 60^{\circ}\right)$			
	Finds l	ength of PT			
		For finding length <i>PT</i> : e.g.,			
	M1	$\tan\left(\frac{\pi}{3}\right) = \frac{PT}{18} \Rightarrow PT = 18\tan\left(\frac{\pi}{3}\right) = \dots$			
		The given values must be used correctly			
	A1	For $PT = 18\sqrt{3}$			
	M1	For the area of $OPTQ = 2 \times \frac{1}{2} \times 18 \times '18 \sqrt{3}' = (561.18)$			
		Their $18\sqrt{3}$ must come from an attempt at using trigonometry.			
	Method				
	Finds le	engths PQ and TO			
M1   '		trigonometry. e.g., $PQ = \sqrt{18^2 + 18^2 - 2 \times 18 \times 18 \cos\left(\frac{2\pi}{3}\right)} = \dots$ and			
		$TO = \frac{\cos\left(\frac{\pi}{3}\right)}{18} = \dots$ The given values must be used correctly			
	<b>A1</b>	For both correct lengths: $PQ = 18\sqrt{3}$ and $TO = 36$			
	M1	For the area of $OPTQ = \frac{1}{2} \times '18\sqrt{3}' \times '36' = (561.18)$ Their $18\sqrt{3}$ and 36 must come from an attempt at using trigonometry			
		uigonomeny			
	M1	For the area of sector $OPQ = \frac{1}{2} \times 18^2 \times \frac{2\pi}{3} = (339.29)$			
		or $\frac{120^{\circ}}{360^{\circ}} \times \pi \times 18^2 = (339.29)$			
	M1	For area of $OPTQ$ – area of Sector $OPQ$ 561.18 - 339.29 = 221.887			
	A1	For 222 cm <sup>2</sup> (must be 3sf) (Units are not required)			
	***	1 of 222 cm (must be 351) (cmis are not required)			

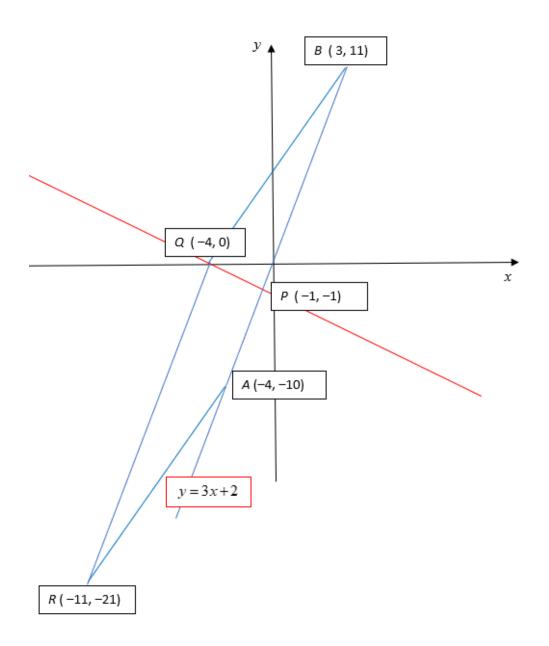
Question number	Scheme	Marks
4 (a)	Gradient = $\frac{11+10}{3+4}$ = 3	
	y+10=3(x+4) or $y-11=3(x-3)$ oe	M1A1 (2)
(b)	e.g. $\left(\frac{4 \times -4 + 3 \times 3}{3 + 4}, \frac{4 \times -10 + 3 \times 11}{3 + 4}\right) = (-1, -1)$	M1 A1 (2)
ALT (b)	Using Vectors $\begin{pmatrix} -4 \\ -10 \end{pmatrix} + \frac{3}{7} \begin{pmatrix} 7 \\ 21 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}  \text{or}  \begin{pmatrix} 3 \\ 11 \end{pmatrix} - \frac{4}{7} \begin{pmatrix} 7 \\ 21 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$	{M1} {A1}
(c)	$-\frac{1}{3} = \frac{n+1}{m+1} \Longrightarrow -\frac{1}{3}(m+1) = n+1$	M1
	$\left(\sqrt{10}\right)^2 = \left(m+1\right)^2 + \left(n+1\right)^2$	M1
	$10 = (m+1)^2 + \frac{1}{9}(m+1)^2$	M1
	$9 = (m+1)^2$ $m = -4 \qquad n = 0$	M1 A1 A1
ALT (c)	Using Vectors $\overrightarrow{AB} = \begin{pmatrix} 7 \\ 21 \end{pmatrix} \text{ so perpendicular to } \overrightarrow{AB} = \begin{pmatrix} 21 \\ -7 \end{pmatrix}$	(6) {M1}
	$\left  \overrightarrow{AB} \right  = 7\sqrt{10} \Rightarrow, \left  \overrightarrow{AP} \right  = 3\sqrt{10}$	{M1,M1}
	$\overrightarrow{PQ} = \frac{\sqrt{10}}{7\sqrt{10}} \times \begin{pmatrix} 21\\ -7 \end{pmatrix} = \begin{pmatrix} 3\\ -1 \end{pmatrix}$	{M1}
	So $Q = (-1 - 3, -11)$	{A1}
	Q = (-4,0)	{A1}
(d)(i)	$AB = \sqrt{(3+4)^2 + (11+10)^2} = 7\sqrt{10}$	M1
	$RQ = \sqrt{(-11+4)^2 + (-21)^2} = 7\sqrt{10}$	A1
(d)(ii)	Gradient of $RQ = \frac{-21 - 0}{11 + 4} = 3$	M1
	-11 + 4 So Gradient of $AB$ (=3) = Gradient of $RQ$	A1 (4)

ALT (d)	Using Vectors	
	$\overrightarrow{RQ} = \begin{pmatrix} -4 - (-11) \\ 0 - (-21) \end{pmatrix} = \begin{pmatrix} 7 \\ 21 \end{pmatrix}$	{M1} {A1}
	$\overrightarrow{AB} = \begin{pmatrix} 7 \\ 21 \end{pmatrix} = \overrightarrow{RQ}$	{M1}
	Because the vectors are the same they must be parallel and the	{A1}
(a)	same length	M1 A1
(e)	$Area = 7\sqrt{10} \times \sqrt{10} = 70$	M1 A1 (2)
ALT (e)	Using Vectors	
	Using Vectors $ \frac{1}{2} \begin{vmatrix} 3 & -4 & -11 & -4 & 3 \\ 11 & 10 & -21 & 0 & 11 \end{vmatrix} $	{M1}
	= 70	{A1}
	Total is	16 marks

Part	Mark	Guidance			
(a)	M1	For a fully correct method of finding an equation of a straight line.			
		$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow \frac{y - (-10)}{11 - (-10)} = \frac{x - (-4)}{3 - (-4)}$			
		$\frac{1}{y_2 - y_1} - \frac{1}{x_2 - x_1} \rightarrow 11 - (-10) - 3 - (-4)$			
		Or finds gradient $\frac{11+10}{3+4} = 3$ and uses $y+10=3(x+4)$ or $y-11=3(x-3)$			
		If $y = mx + c$ is used, they must find a complete equation for this mark.			
		Allow <b>one</b> error only for the award of this mark.			
	<b>A1</b>	For a correct line in any form.			
		y+10=3(x+4) or $y-11=3(x-3)$			
		or $y = 3x + 2$			
		or even $\frac{y+10}{21} = \frac{x+4}{7}$ but do not allow incomplete processing.			
<b>(b)</b>	M1	For one correct from $x = -1$ or $y = -1$			
	A1	For the correct coordinates of point $P(-1,-1)$			
		Accept $x = -1$ $y = -1$			
(c)	M1	Uses the perpendicular gradient to set up an equation in $m$ and $n$ .			
		$-\frac{1}{3} = \frac{n - (-1)}{m - (-1)} \Rightarrow -\frac{1}{3}(m+1) = n+1 \text{ or } n = -\frac{1}{3}n + \frac{4}{3}$			
		$3' - m - (-1)' \rightarrow 3' + (m+1) - m + 1 + 0! + m - 3'' + 3$			
		Ft their gradient in part (a) and their <i>P</i> from part (b) for this mark.			
	<b>M1</b>	Uses Pythagoras theorem to set up an equation in $m$ and $n$ .			
		$\left(\sqrt{10}\right)^2 = \left(m - '(-1)'\right)^2 + \left(n - '(-1)'\right)^2$			
		Ft their coordinates of point <i>P</i> form part (b) for this mark.			
	M1	Attempts to solve their two equations in <i>n</i> and <i>m</i> simultaneously and forms a			
		quadratic equation in one variable only.			
		$10 = (m+1)^{2} + \frac{1}{9}(m+1)^{2} \Rightarrow 9 = (m+1)^{2} \text{ or } 0 = m^{2} + 2m - 8$			
		or $10 = 9(n+1)^2 + (n+1)^2 \Rightarrow 0 = 10n^2 + 20n$			
	M1	For solving their either: $9 = (m+1)^2 \Rightarrow m =$ or $0 = 10n^2 + 20n \Rightarrow n =$			
		which must be a quadratic equation.			
	<b>A1</b>	For finding either $m = -4$ or $n = 0$			
	A1	Condone the sight of $m = 2$ for this mark.			
		For finding <b>both</b> $m = -4$ <b>and</b> $n = 0 \Rightarrow (-4, 0)$			
	AIT .	The final answer must be given as coordinates.			
(4)(2)	ALI -	using vectors – see main scheme.			
(d)(i)	М1	For finding <b>either</b> the length $AB = \sqrt{(3+4)^2 + (11+10)^2} = 7\sqrt{10}$			
	M1	Or $RQ = \sqrt{(-11+4)^2 + (-21)^2} = 7\sqrt{10}$			
	<b>A1</b>	For finding <b>both</b> the length $AB = \sqrt{(3+4)^2 + (11+10)^2} = 7\sqrt{10}$			
	111	And $RQ = \sqrt{(-11+4)^2 + (-21)^2} = 7\sqrt{10}$ and states they are equal			
(d)(ii)	M1	The gradient of $RQ = \frac{-21 - '0'}{-11 - '(-4)'} = '3'$			
	Ft their coordinates from part (c)				

	A1 States that the gradient of $RQ$ = gradient of $AB$ [from (a)]					
	<b>A1</b>	States that the gradient of $NQ$ – gradient of $AD$ [from (a)]				
	ALT ·	– Uses vectors, see main scheme. Ft their coordinates of $Q - (m, n)$				
(e)	M1	For a correct expression for the area using their length of AB and the given				
		length of $PQ\left(\sqrt{10}\right)$				
		Area = $7\sqrt{10} \times \sqrt{10} =$				
	<b>A1</b>	For the area = 70 [square units]				
	ALT	T – Uses the discriminant				
	<b>M1</b>	For a correct expression of the area in sequential order using their				
		coordinates for Q				
		Area = $\frac{1}{2}\begin{vmatrix} 3 & -4 & -11 & '-4' & 3 \\ 11 & 10 & -21 & '0' & 11 \end{vmatrix}$				
	A 1					
T. 6 1		A1   Area = 70 [square units]				

**Useful sketch** 



Question number	Scheme	Marks
5 (a)	$u_2 + u_4 = ar + ar^3 = 212.5$	M1
	$u_3 + u_4 = ar^2 + ar^3 = 62.5$ $(1+r^2)  17$	
	$\frac{\left(1+r^2\right)}{\left(r+r^2\right)} = \frac{17}{5}$	M1
	$12r^2 + 17r - 5 = 0$	M1
	(4r-1)(3r+5) = 0	M1
	$r = \frac{1}{4}$ $r = -\frac{5}{3}$	
	$r = \frac{1}{4}$ $r = -\frac{1}{3}$	<b>A</b> 1
		(5)
(b)	1 $a = 800 - 3200$	
	$r = \frac{1}{4} \Rightarrow a = 800$ So $\frac{a}{1-r} = \frac{800}{3} = \frac{3200}{3}$	M1 A1
	$\frac{1}{4}$	(2)
	Tota	l 7 marks

Part	Mark	Guidance		
(a)	M1	For either $ar + ar^3 = 212.5$ or $ar^2 + ar^3 = 62.5$ correct		
		For either $ar + ar^3 = 212.5$ or $ar^2 + ar^3 = 62.5$ correct For attempting to eliminate a or ar either by division or substitution:		
	M1	e.g. $\frac{ar(1+r^2)}{ar(r+r^2)} = \frac{212.5}{62.5} \Rightarrow \frac{(1+r^2)}{(r+r^2)} = \frac{212.5}{62.5} = \left(\frac{17}{5}\right)$		
	Motho	An attempt involves some factorisation to eliminate <i>a</i> or <i>ar</i> <b>d 1 – finds a 3TQ</b>		
	Memo			
For forming a 3TQ in r only using their expressions. $(12r^2 + 17r - 5 = 0 \text{ oe})$ Accept for example $150r^2 + 212.5r - 62.5 = 0$				
		For an attempt to solve their 3TQ to give <b>two</b> values of $r$		
	dM1	See General Guidance for the definition of an attempt. For example: $(4r-1)(3r+5)=0 \Rightarrow r=,$		
		This mark is dependent on the <b>FIRST</b> M mark being awarded		
	Metho	d 2 – finds a cubic equation		
	M1	For forming a cubic with a common factor of $r$ in each term. e.g. $12r^3 + 17r^2 - 5r = 0$		
		For factorising their cubic equation to achieve $r(12r^2+17r-5)=0$		
	dM1	and for an attempt to solve their 3TQ to give <b>two</b> values of $r$ Ignore $r = 0$ if also given. See General Guidance for the definition of an attempt. For example: $(4r-1)(3r+5) = 0 \Rightarrow r =,$		
		-		
	A1	This mark is dependent on the <b>FIRST</b> M mark being awarded  For the correct values; $r = \frac{1}{4}$ and $r = -\frac{5}{3}$ (reject $r = 0$ if seen earlier)		
(b)		Uses their $r = \frac{1}{r}$ [where $ r  < 1$ ] to find the value of $a$ (800) with the correct		
	M1	Uses their $r = \frac{1}{4}$ [where $ r  < 1$ ] to find the value of $a$ (800) with the correct formula for the sum of a geometric series to infinity. Condone an incorrect value of $a$ even if they have used $r = \frac{1}{4}$ [The formula is given on page 2 of this booklet].		
	$S_{\infty} = \frac{a}{1 - r} = \frac{'800'}{1 - '\frac{1}{4}'} = \dots$ For the correct value, $S_{\infty} = \frac{3200}{3}$ or $1066\frac{2}{3}$			
		Do not accept for example 1066.67 unless the stated value is 1066.6		

Question number	Scheme	Marks
6 (a)	f(3) = 27 + 9p + 9 - 30 + q = 0	M1 A1
	9p + q + 6 = 0 *	A1 cso
		(3)
(b)	$f(-p) = -p^3 + p^2(p+1) + 10p + q = 0$	M1 A1
	$p^2 + 10p + q = 0$ *	A1 cso
		(3)
(c)	$p^2 + 10p - 9p - 6 = 0$	M1
	$p^2 + p - 6 = 0$	A1
	(p+3)(p-2) = 0	M1
	p=2 $q=-24$	A1 A1
		(5)
(d)	(x+a)(x-3)(x+2)	
	So $-3 \times 2 \times a = -24$ $a = 4$	M1
	(x+4)(x-3)(x+2)	A1
		(2)
	Total	13 marks

Part	Mark	Guidance					
Gener	General guidance for marking parts (a) and (b)						
•	• For the award of full marks in parts (a) and/or (b) you <b>must</b> see = 0 used in a line of working <b>before</b> the final answer.						
•	which	ndidate does not use = 0 in either parts (a) or (b) [except in the final line – is a given answer] deduct the M mark (and the subsequent A marks) in only st occurrence of the absence.					
(a)	M1	For using $f(\pm 3) = 0$ in the given equation set = 0					
	A1	For obtaining the correct unsimplified expression: $27 + 9p + 9 - 30 + q = 0$					
	A1	For obtaining the given equation $9p + q + 6 = 0*$					
	cso	Note: This is a show question. There must be no errors seen.					
<b>(b)</b>	M1	For use of $f(\pm p) = 0$ in the given equation set = 0					
	A1	For obtaining the correct unsimplified expression:					
		$-p^3 + p^2(p+1) + 10p + q = 0$					
	<b>A1</b>	For obtaining the correct given equation $p^2 + 10p + q = 0$ *					
	cso	Note: This is a show question. There must be no errors seen.					

(c)	M1	For attempting to solve the given two equations simultaneously to achieve				
		a 3TQ in either p or q only.				
		E.g. substitutes $q = \mp 9p \mp 6$ or $\left[p = \frac{\mp q \mp 6}{9} \text{ and } p^2 = \frac{(\mp q \mp 6)^2}{81}\right]$ into				
		$p^2 + 10p + q = 0$				
		This mark may be implied by the correct 3TQ				
	<b>A1</b>	For the correct 3TQ $p^2 + p - 6 = 0$ or $q^2 + 3q - 504 = 0$				
	M1	For an attempt to solve their 3TQ in either <i>p</i> or <i>q</i> using factorisation, use of the formula or completing the square.  See general guidance for the definition of an attempt.  For example:				
		$(p+3)(p-2) = 0 \Rightarrow p = \dots$ or $(q+24)(q-21) = 0 \Rightarrow q = \dots$ ,				
		If a candidate uses their calculator to solve their 3TQ, the final values must be correct for the award of this mark unless a valid method is seen.				
	A1	For either the correct value of $p$ OR the correct value of $q$				
		p = 2  or  q = -24				
		Condone the presence $p = -3$ , and/or $q = 21$				
	A1	For both the correct value of $p = 2$ AND the correct value of $q = -24$ Must reject $p = -3$ , and/or $q = 21$ if seen.				
(d)	M1	[f(x) = (x+a)(x-3)(x+2')]				
, ,		For attempting to find the value of a				
		$-3 \times 2' \times a = -24' \Rightarrow a = \dots$				
		OR				
		For an attempt using division with their values of p and q				
		$x^2 - x - 6$ $x^3 + 3x^2 - 10x - 24$				
		$(x^2-x-6)x^3+3x^2-10x-24$				
		Allow a quotient of $x+b$ where $b$ is a constant.				
	A1	For the correct factorised expression $(x+4)(x-3)(x+2)$ which must be				
		written out in full on one line.				

Question number			Scheme		Marks
7 (a)	2	3	4		B1 B1
	3.73	4.28	5		(2)
(b)	Points plotte	d			B1ft
	Joined up wi	th a smooth o	curve		B1ft
					(2)
(c)	$\log_{3}(6-2x)$ $6-2x = 3^{\frac{x}{4}}$ $8-2x = 3^{\frac{x}{4}}$ $y = 8-2x$	$=\frac{x}{4}$			M1
	$6 - 2x = 3^{\frac{x}{4}}$				M1
	$8 - 2x = 3^{-4}$	+ 2			A1
	y = 8 - 2x	drawn			M1
	x = 2.1				A1
					(5)
				Tota	d 9 marks

Part	Mark	Guidance					
(a)	B1	For two points (rounded correctly) correct from;					
						1	
		0 1	2	3	4	5	
		3 3.32 3	3.73	4.28	5	5.95	
	B1	All three points correct and	correctly	z rounded			
	DI		All three points correct and correctly rounded. Penalise rounding only once here. Condone 5.00				
(b)	B1ft	All points plotted within half of one square. Ft <b>their</b> values of y					
(~)	2220	for $x = 2,3,4$ respectively					
	B1ft		All drawn points joined up in a smooth curve				
		1	, ,				
			/				
		+		<b>→</b>			
		-					
(c)	M1	For use of power law to obta	For use of power law to obtain $\log_3(6-2x) = \frac{\pm x}{4}$				
	IVII	Tor use or power law to obta	iii 10g <sub>3</sub>	(0-2x)=	4		
		+x					
	<b>M1</b>	For removing the $\log_3$ to obtain: $6-2x=3^{\frac{3}{4}}$					
		Allow $(6-2x)^4 = 3^{\pm x}$ for this mark.					
	A1	$_{ m Y}$ $_{ m T}$					
		For obtaining the equation $8-2x=3^{\frac{1}{4}}+2$ or $(eg., -2x+8=2+3^{\frac{1}{4}})$					
			For drawing their straight line, provided it is of the form $y = k - 2x$ where $k$				
		is a constant.					
				,			
	<b>M1</b>						
				H			
		[Check coordinates (1, 6) (	2, 4) (	3, 2) (4, 0)	)]		
	A 1	For the intersection point (	r _) 2 1				
	A1	For the intersection point (	$\lambda - j = 2.1$				

Question number	Scheme	Marks
8	$\log_A a + 2\log_A b = \frac{5}{2}$	M1
	$\log_4 a + 2\log_4 b = \frac{5}{2}$ $\log_4 (ab^2) = \frac{5}{2}$ $32 = ab^2$ $2^a = \frac{2^{16}}{2^{2b^2}}$	M1
	$32 = ab^2$	A1
	$2^a = \frac{2^{16}}{2!^2}$	M1
	$a = 16 - 2b^2$ or $b^2 = 8 - \frac{1}{2}a$	A1
	$32 = a(8 - \frac{1}{2}a)$ or $32 = (16 - 2b^2)b^2$	M1
	$a^2 - 16a + 64 = 0$ or $2b^4 - 16b^2 + 32 = 0$	A1
	a=8 $b=2$	A1
	10ta	l 8 marks

Mark	Guidance
Log eq	
M1	For an attempt to change the base of $3\log_8 b$ to base 4 using $\log_a x = \frac{\log_b x}{\log_b a}$ $3\log_8 b = \frac{3\log_4 b}{\log_4 8} = \frac{3\log_4 b}{\frac{3}{2}} = 2\log_4 b  \text{[accept } p\log_4 b \text{ where } p \neq 3\text{]}$
	2
M1	Uses $n \log A = \log A^n$ and $\log A + \log B = \log AB$ to combine the logs correctly
	$\log_4(ab^2) = \frac{5}{2}$ [ft their <i>p</i> provided $p \neq 1$ ]
<b>A1</b>	For removing the logs in the equation to obtain $32 = ab^2$ o.e. e.g. $a^2b^4 = 1024$
	Method 2 – Works in base 8
N/1	For an attempt to change the base of $\log_4 a$ to base 8 using $\log_a x = \frac{\log_b x}{\log_b a}$
M1	$\log_4 a = \frac{\log_8 a}{\frac{2}{3}} = \frac{3\log_8 a}{2}$ [accept $q \log_8 a$ where $q \neq 1$ ]
	Uses $n \log A = \log A^n$ and $\log A + \log B = \log AB$ correctly to combine the logs
M1	$\log_8(a^{\frac{3}{2}}b^3) = \frac{5}{2} \qquad \text{[ft their } q\text{]}$
	For removing the logs in the equation to obtain $a^{\frac{3}{2}}b^3 = 8^{\frac{5}{2}}$ and rearranges (raises
A1	both sides to the power of $\frac{2}{3}$ ) to obtain $32 = ab^2$
Second	equation
	For attempting to change the second equation to powers of 2 or 4:
M1	$2^{a} = \frac{2^{16}}{2^{2b^{2}}} \Longrightarrow \left[2^{a} = 2^{\left(16 - 2b^{2}\right)}\right] \text{ or } 4^{\frac{a}{2}} = \frac{4^{8}}{4^{b^{2}}} = \left(4^{\frac{a}{2}} = 4^{8 - b^{2}}\right)$
	At least one correct change of term e.g either $2^{16}$ or $2^{2b^2}$ OR either $4^{\frac{a}{2}}$ or $4^8$
<b>A1</b>	Combines the powers to achieve $a = 16 - 2b^2$ or $\frac{a}{2} = 8 - b^2$ oe
Attemp	t to solve the simultaneous equations
	For an attempt to solve their equations simultaneously, both of which must be in terms of $a$ and $b^2$ , to obtain a 3TQ in either $a$ or $b^2$ .
M1	
	$32 = a(8 - \frac{1}{2}a) \Rightarrow a^2 - 16a + 64 = 0$ or $32 = (16 - 2b^2)b^2 \Rightarrow 2b^4 - 16b^2 + 32 = 0$
	For an attempt to solve their 3TQ in either $a$ or $b^2$ by any method. See General Guidance for the definition of an attempt
M1	For example:
IVII	$a^{2} - 16a + 64 = 0 \Rightarrow (a - 8)(a - 8) = 0 \Rightarrow a =$
	$2b^{4} - 16b^{2} + 32 = 0 \Rightarrow b^{4} - 8b^{2} + 16 = 0 \Rightarrow (b^{2} - 4)(b^{2} - 4) = 0 \Rightarrow b = \dots$
A1	For $a = 8$ and $b = 2$ [If $b = \pm 2$ is given as the final answer, withhold this mark].

Question number	Scheme	Marks
9 (a)	$\frac{\mathrm{d}A}{\mathrm{d}t} = 0.03$	В1
	$A = \frac{1}{2}x^2 \sin 60^\circ = \frac{\sqrt{3}}{4}x^2$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{\sqrt{3}}{2}x$	A1
	When $x = 2$ $\frac{dx}{dt} = \frac{1}{\sqrt{3}} \times 0.03 = 0.0173 \text{ cm/s}$	M1 A1 (5)
(b)	$V = \sqrt{3}x^3 \qquad \frac{\mathrm{d}V}{\mathrm{d}x} = 3\sqrt{3}x^2$	M1
	When $x = 2 \frac{dV}{dt} = 12\sqrt{3} \times 0.0173 = 0.36$	M1 A1 (3)
	Tota	l 8 marks

Part	Mark	Guidance			
(a)	B1	For stating or using correctly in their Chain Rule $\frac{dA}{dt} = 0.03$			
		For using the <b>correct</b> formula $\left(\frac{1}{2}ab\sin C\right)$ with the correct lengths and			
		angle of $60^{\circ}$ or $\frac{\pi}{3}$ , for the cross-sectional area of the prism to			
	M1	obtain $A = \frac{1}{2}x^2 \sin 60^\circ = \left(\frac{\sqrt{3}}{4}x^2\right)$ and differentiating their expression which			
		must be as a minimum $A = px^2$ to obtain $\frac{dA}{dx} = qx$ [where $p$ and $q$ are constants].			
		[The height of the triangle is $\frac{\sqrt{3}}{2}x$ if they use $\frac{1}{2} \times \text{base} \times \text{height}$ ]			
	<b>A1</b>	For the correct $\frac{dA}{dx} = \frac{\sqrt{3}}{2}x$			
	M1	For applying a <b>correct</b> Chain rule using their $\frac{dA}{dx}$ and $x = 2$ to obtain $\frac{dx}{dt} = \left(\frac{1}{\frac{dA}{dx}} \times \frac{dA}{dt}\right) = \frac{dx}{dA} \times \frac{dA}{dt} = \frac{2}{\sqrt{3}} \times \frac{1}{2} \times 0.03$			
	<b>A1</b>	$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.0173$			
(b)	M1	For a correct expression for the volume using their A from part (a) to obtain $V = \frac{\sqrt{3}}{4}x^2 \times 4x = \left(\sqrt{3}x^3\right)$ and differentiating their expression which must be as a minimum $V = mx^3$ to obtain as a minimum $\frac{dV}{dx} = nx^2$ [where m and n are constants] $\left(\frac{dV}{dx} = 3\sqrt{3}x^2\right)$			
	M1	For applying a correct Chain rule using their $\frac{dV}{dx}$ and $x = 2$ to obtain $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 12\sqrt{3} \times 0.0173 = \begin{bmatrix} 0.359 \end{bmatrix}  \text{(ft their } \frac{dx}{dt} \text{)}$ Note: $\frac{dx}{dt} = 0.0173 \text{ or } \frac{\sqrt{3}}{100}$ $\left(\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 12\sqrt{3} \times \frac{\sqrt{3}}{100} = \frac{9}{25} = 0.36\right)$			
	A1	For awrt 0.36			

Question number	Scheme	Marks
10 (a)	$x = \tan^{-1}(-3) = -72$	M1
	$x = 108 \qquad x = 288$	A1 A1
		(3)
(b)	$7\sin^2\theta + \sin\theta\cos\theta = 6(\sin^2\theta + \cos^2\theta)$	M1
	$\sin^2\theta + \sin\theta\cos\theta - 6\cos^2\theta = 0$	
	$\sin^2\theta \sin\theta$	
	$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin\theta}{\cos\theta} - 6 = 0$	M1
	$\tan^2\theta + \tan\theta - 6 = 0$	A1 cso
		(3)
(c)	$(\tan y + 3)(\tan y - 2) = 0$	M1
	$\tan y = -3 \qquad \tan y = 2$	A1
	y = 108,288 $y = 63,243$	A1ft A1
		(4)
	Total	10 marks

Part	Mark	Guidance	
(a)		For using inverse tan to obtain any correct angle	
	M1	$\tan^{-1}(-3) \Rightarrow x = -71.565^{\circ}$ Accept awrt 72 °	
	<b>A1</b>	For either 108 or 288	
	A1	For <b>both</b> 108 <b>and</b> 288	
<b>(b)</b>	Uses $\sin^2 \theta + \cos^2 \theta = 1$ on the given equation to obtain		
	IVII	$7\sin^2\theta + \sin\theta\cos\theta = 6(\sin^2\theta + \cos^2\theta)$	
	M1	For rearranging and dividing through by $\cos^2 \theta$ with the $\frac{\sin \theta}{\cos \theta} = \tan \theta$	
	WII	identity to obtain a 3TQ: $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\sin \theta}{\cos \theta} - 6 = 0 \Rightarrow \left(\tan^2 \theta + \tan \theta - 6 = 0\right)$	
	ALT		
	M1	Divides the given equation through by $\cos^2 \theta$ with the $\frac{\sin \theta}{\cos \theta} = \tan \theta$	
	1,11	identity to obtain $7 \tan^2 \theta + \tan \theta = \frac{6}{\cos^2 \theta}$	
	3.54	Uses $\sin^2 \theta + \cos^2 \theta = 1$ to obtain $\tan^2 \theta + 1 = \frac{1}{\cos^2 \theta}$ and uses this result on	
	M1	the given equation and rearranges to achieve a 3TQ to obtain	
		$7 \tan^2 \theta + \tan \theta = 6 \left( 1 + \tan^2 \theta \right) \Longrightarrow \left( \tan^2 \theta + \tan \theta - 6 = 0 \right)$	
	A1	For obtaining the given expression $\tan^2 \theta + \tan \theta - 6 = 0$ * in full.	
	cso	Note: This is a show question, there must be no errors in the solution.	
(c)	M1	For changing $7\sin^2 y + \sin y \cos y = 6$ to $\tan^2 y + \tan y - 6 = 0$ [this step	
	IVII	must be correct] and then attempting to solve the 3TQ by any method.	
	<b>A1</b>	For $\tan y = -3$ and $\tan y = 2$	
	A1ft	For <b>both</b> $y = 108$ <b>and</b> 288 (ft from (a))	
	A1	For <b>both</b> $y = 63$ <b>and</b> 243	
Roun	ding orr	Pars. Penalise rounding only once in this question when first seen provided	

**Rounding errors:** Penalise rounding only once in this question when first seen provided angles round to 108, 288, 63 or 243

Question number	Scheme	Marks
11	$e^x = \frac{4}{e^x} \Longrightarrow e^{2x} = 4$	M1
	$x = \frac{1}{2} \ln 4$ or $x = \ln 2$	A1
	$\pi \int_0^{\ln 2} e^{2x} dx + \pi \int_{\ln 2}^a 16e^{-2x} dx$	M1 M1
	$\pi \left[ \frac{1}{2} e^{2x} \right]_{0}^{\ln 2} + \pi \left[ -8 e^{-2x} \right]_{\ln 2}^{a}$	M1
	$\pi \left(2 - \frac{1}{2}\right) + \pi \left(-8e^{-2a} + 2\right) =$	M1
	$\frac{7\pi}{2} - 8\pi e^{-2a} \qquad a = 2 \text{ and } k = \frac{7\pi}{2}$	M1A1
Total 8 mark		l 8 marks

Mark	Guidance
M1	Sets $e^x = \frac{4}{e^x}$ and attempts to make x the subject
	A minimum of $e^{2x} = 4$ or $e^x = 2$ is required to be seen for this mark.
A1	For obtaining either $x = \frac{1}{2} \ln 4$ or $x = \ln 2$
M1	For stating $\pi \int_0^{\ln 2} (e^x)^2 dx = \pi \int_0^{\ln 2} e^{2x} dx$ using the limits of their ln 2 (which must
	be of the form $\ln k$ ) and 0 correctly
	<b>Note:</b> Condone a missing $\pi$ here if it is seen at the final M mark.
M1	For stating $\pi \int_{\ln 2}^{a} (4e^{-x})^2 dx = \pi \int_{\ln 2}^{a} 16e^{-2x} dx$ using a and the limit of their ln 2
	(which must be of the form $\ln k$ where $k$ is consistent between the two integrals)
	correctly <b>Note:</b> Condone a missing $\pi$ if it is seen at the final M mark.
M1	For an attempt to integrate <b>both</b> expressions obtaining:
	Either $\left  \frac{e^{2x}}{2} \right $ or $\left  \frac{-16e^{-2x}}{2} \right $ (condone $\left  \frac{16e^{-2x}}{2} \right $ )
	For this mark ignore the absence of $\pi$ or incorrect/absent limits
	[need not be simplified]
dM1	For substituting <b>their</b> limits correctly (where their ln 2 must be of the form ln k
	where $k$ is consistent between the two integrals) into their integrated expression. For this mark ignore the absence of $\pi$
	$\pi \left( \frac{e^{2\ln 2}}{2} - \frac{e^0}{2} \right) + \pi \left( -16e^{-2a} - \frac{\left( -16e^{-2\ln 2} \right)}{2} \right) = \pi \left( 2 - \frac{1}{2} \right) + \pi \left( -8e^{-2a} + 2 \right)$
	This mark is dependent on the previous M mark.
M1	For equating to the given expression for the volume and equating coefficients to
	find values for a and k. We must see $\pi$ used here for the award of this mark.
	$\left(\pi\left(2-\frac{1}{2}\right)+\pi\left(-8e^{-2a}+2\right)=\frac{7\pi}{2}-8\pi e^{-2a}\right)$
	$7\pi \qquad 8\pi e^{-2a} - k \qquad 8\pi e^{-4} \rightarrow k - a - a$
	$\frac{7\pi}{2} - 8\pi e^{-2a} = k - 8\pi e^{-4} \Rightarrow k =,  a =$
<b>A1</b>	For $a=2$ and $k=\frac{7\pi}{2}$
	۷