Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel International GCSE**

Centre Number

Candidate Number

# Thursday 18 June 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02R**

# Further Pure Mathematics

## Paper 2R

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

### Series

**Arithmetic series**

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

**Geometric series**

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

### Calculus

**Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

### Trigonometry

**Cosine rule**

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

### Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

**Answer all TEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** The *n*th term of an arithmetic series $A$ is $a_n$

The *n*th term of a geometric series $G$ is $t_n$

For these two series

$$a_1 = t_1 \qquad a_{10} = t_3 = 48 \qquad a_{10} = 4t_2$$

Find

(i) the common ratio of $G$,

(ii) the common difference of $A$.

**(6)**

**(Total for Question 1 is 6 marks)**

**2** $f(x) = x^3 + px + q$ where $p$ and $q$ are constants.

The remainder when $f(x)$ is divided by $(x - 1)$ is $-12$

The remainder when $f(x)$ is divided by $(x - 4)$ is 30

(a) Find the value of $p$ and the value of $q$.

**(6)**

Using your values of $p$ and $q$

(b) show that $f(3) = 0$

**(1)**

(c) Express $f(x)$ as a product of linear factors.

**(3)**

(d) Hence solve the equation $f(x) = 0$

**(1)**

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**Question 2 continued**

**(Total for Question 2 is 11 marks)**

**3**

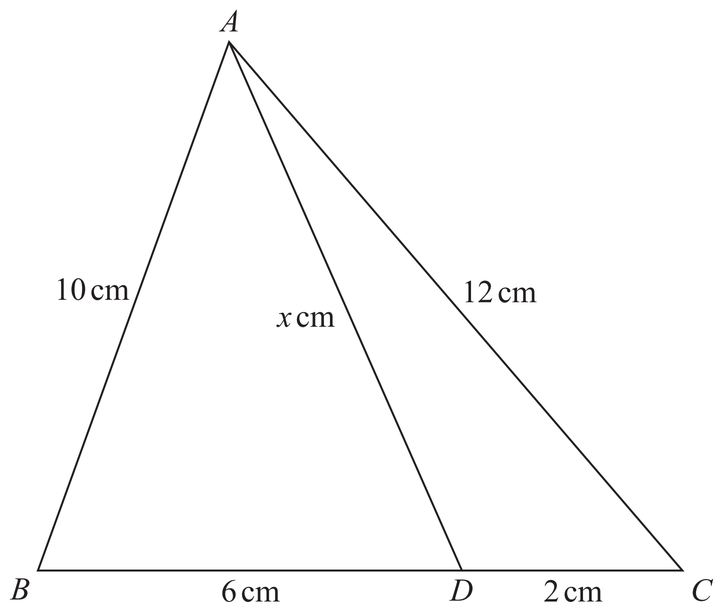


Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows the triangle $ABC$ in which $AB = 10$ cm and $AC = 12$ cm.

The point $D$ lies on $BC$ such that $BD = 6$ cm, $DC = 2$ cm and $AD = x$ cm.

(a) Show that $x = 11$

**(4)**

(b) Find the area, in $cm^2$ to 3 significant figures, of triangle $ADB$.

**(4)**

**Question 3 continued**

**(Total for Question 3 is 8 marks)**

**4** (a) Complete the table of values for $y = 2x + 1 + \frac{2}{x^2}$

Give your answers to 2 decimal places where appropriate.

| $x$ | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 |
|---|---|---|---|---|---|---|---|
| $y$ | | 5 | | | 6.32 | | 8.16 |

**(2)**

(b) On the grid opposite, draw the graph of $y = 2x + 1 + \frac{2}{x^2}$ for $0.5 \leqslant x \leqslant 3.5$

**(2)**

(c) Use your graph to obtain estimates, to 1 decimal place, of the roots of the equation

$$2x + \frac{2}{x^2} = 7 \quad \text{in the interval } 0.5 \leqslant x \leqslant 3.5$$

**(2)**

(d) By drawing a suitable straight line on the grid, obtain estimates, to 1 decimal place, of the roots of the equation

$$\frac{3x}{2} + \frac{2}{x^2} = 5 \quad \text{in the interval } 0.5 \leqslant x \leqslant 3.5$$

**(5)**

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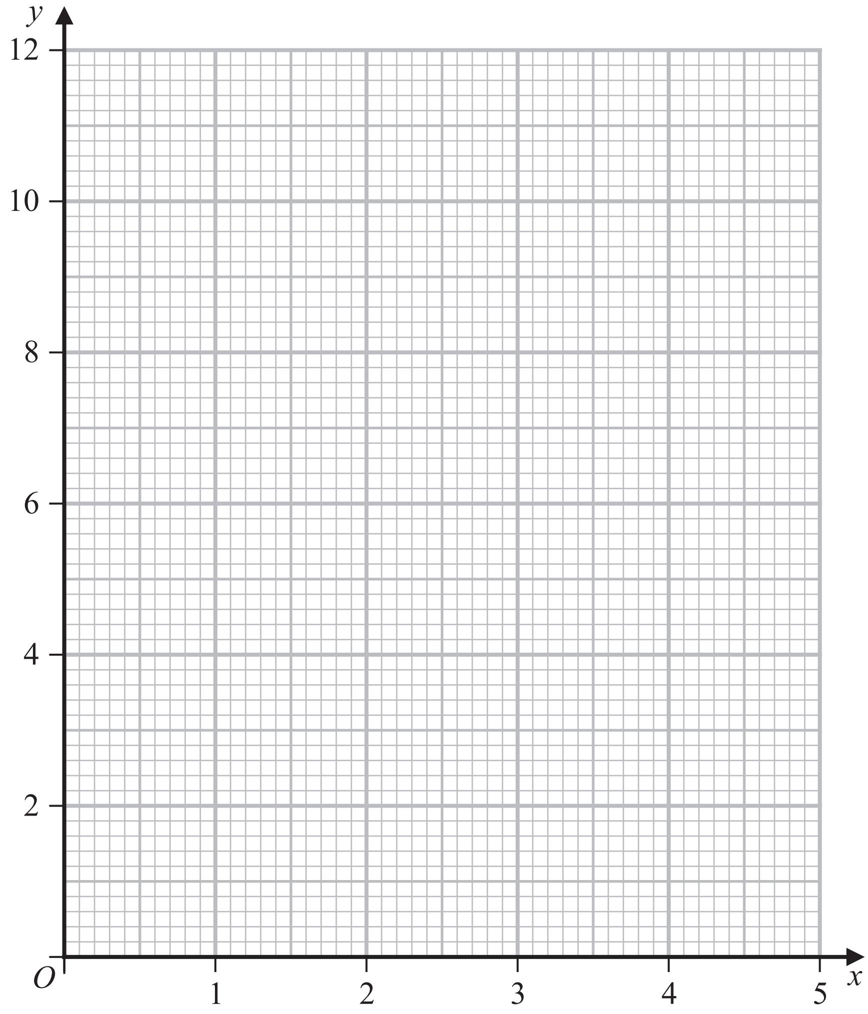
**Question 4 continued**




Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

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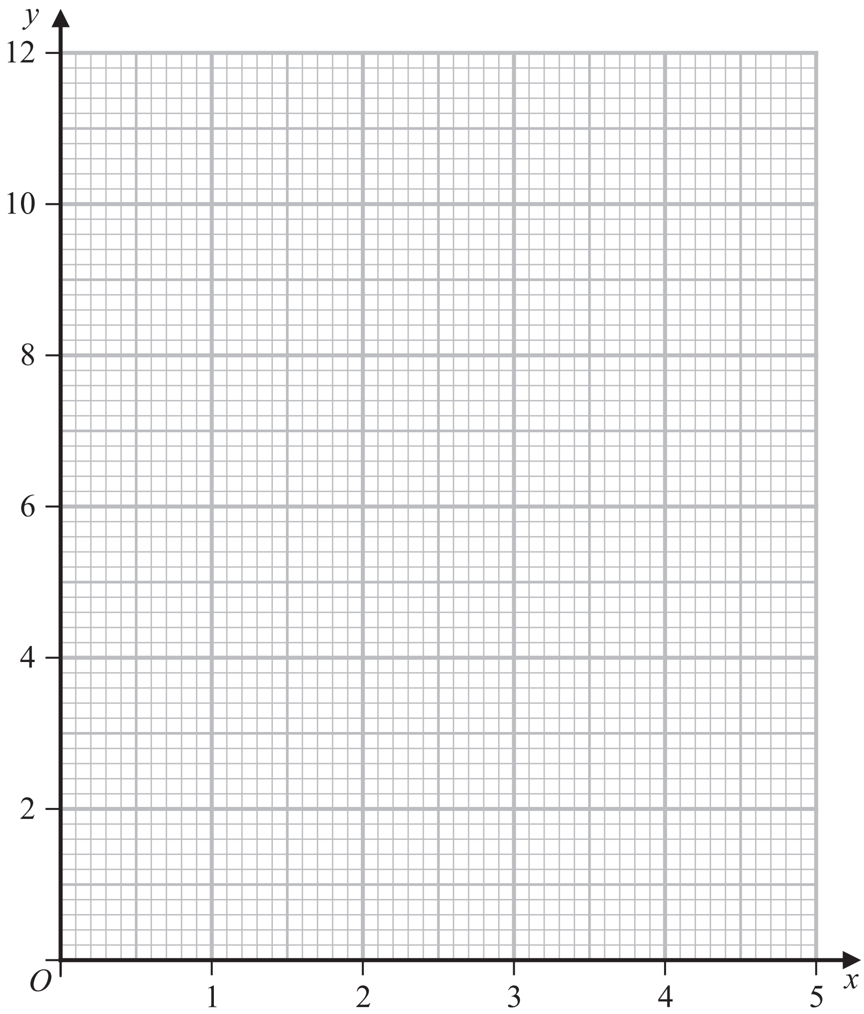
**Question 4 continued**

**Only use this grid if you need to redraw your graph.**




**(Total for Question 4 is 11 marks)**



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*Turn over*

**5**

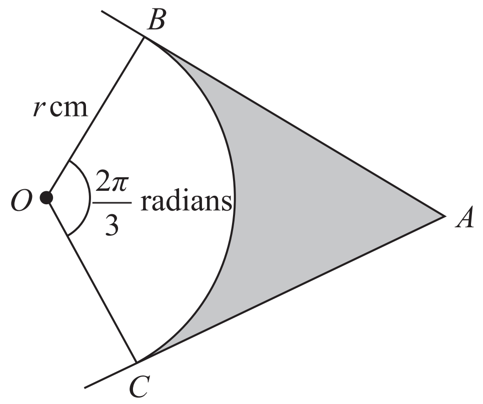


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**Figure 2**

In Figure 2, $AB$ and $AC$ are tangents to a circle with centre $O$ and radius $r$ cm.

The points $B$ and $C$ lie on the circle so that $OBC$ is a sector of this circle and $\angle BOC = \frac{2\pi}{3}$ radians.

Given that the area of the shaded region is $10\,\text{cm}^2$,

find, to 3 significant figures, the value of $r$.

**(8)**

Question 5 continued

(Total for Question 5 is 8 marks)

**6**

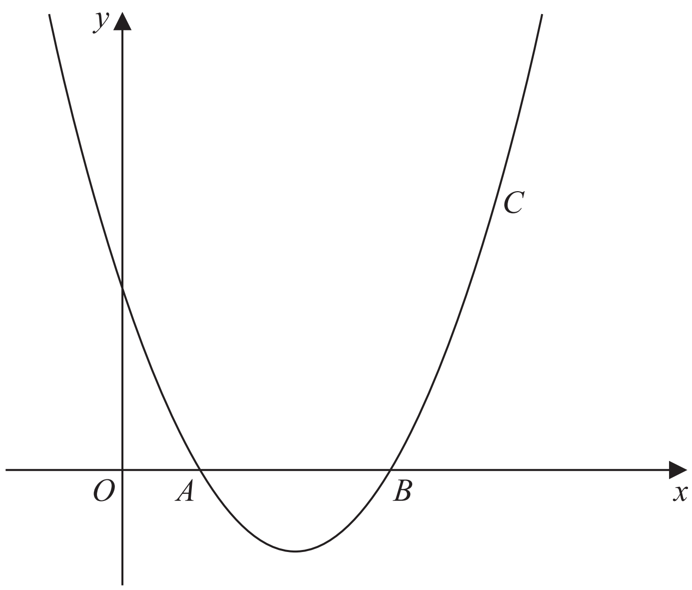


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**Figure 3**

The curve $C$ with equation $y = x^2 - 5x + 4$ crosses the $x$-axis at the points $A$ and $B$, as shown in Figure 3

(a) Find the coordinates of $A$ and the coordinates of $B$.

**(3)**

The tangent to $C$ at $A$ meets the tangent to $C$ at $B$ at the point $T$.

(b) Find the coordinates of $T$.

**(6)**

The normal to $C$ at $A$ meets the normal to $C$ at $B$ at the point $N$.

(c) Find the coordinates of $N$.

**(3)**

(d) Find the area of the quadrilateral $ATBN$.

**(3)**

**Question 6 continued**

Question 6 continued

**Question 6 continued**

**(Total for Question 6 is 15 marks)**

**7** (a) Find the set of values of $k$ for which the equation $kx^2 - 4x + 2k = 7$ has real roots

**(4)**

Given that the roots of the equation $kx^2 - 4x + 2k = 7$ are $\alpha$ and $\beta$,

(b) form a quadratic equation with roots $\dfrac{\alpha + 1}{\alpha}$ and $\dfrac{\beta + 1}{\beta}$

Give each coefficient in terms of $k$.

**(8)**

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**Question 7 continued**



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**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 12 marks)**



P 6 2 2 8 4 A 0 2 1 3 2

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**8** Solve the equation $\log_3 x - 2\log_x 3 = 1$

**(7)**

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**Question 8 continued**

**(Total for Question 8 is 7 marks)**



P 6 2 2 8 4 A 0 2 3 3 2

**9** Given that

$$x = e^{-t} \sin 2t$$

show that

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$$

**(8)**

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**Question 9 continued**

**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 8 marks)**



P 6 2 2 8 4 A 0 2 7 3 2

**10** $$f(x) = 32x^3 - 33x + 1$$

(a) Show that $f(1) = 0$

**(1)**

(b) Hence using an algebraic method solve $f(x) = 0$

**(4)**

Diagram **NOT** accurately drawn

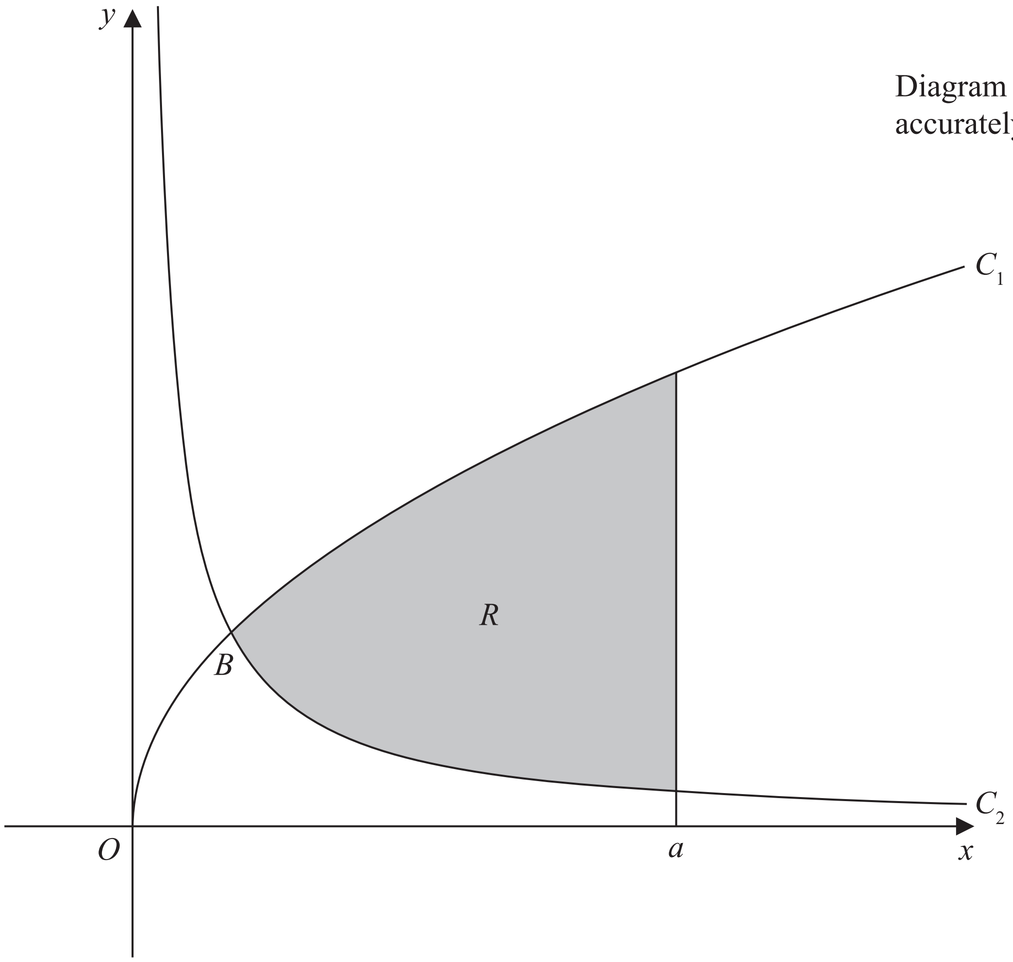


**Figure 4**

The region $R$, shown shaded in Figure 4, is bounded by the curve $C_1$ with equation $y = \sqrt{x}$, by the curve $C_2$ with equation $y = \frac{1}{8x}$ and by the line with equation $x = a$

The curves $C_1$ and $C_2$ intersect at the point $B$, with $x$ coordinate $p$, where $p < a$

(c) Find the value of $p$.

**(2)**

The region $R$ is rotated through 360° about the $x$-axis to generate a solid with volume $\frac{27\pi}{64}$

(d) Use algebraic integration to find the value of $a$.

**(7)**

**Question 10 continued**

**Question 10 continued**

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Question 10 continued

**Question 10 continued**

**(Total for Question 10 is 14 marks)**

**TOTAL FOR PAPER IS 100 MARKS**